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Using a Fuzzy Piecewise Regression Analysis to Predict the Nonlinear Time-Series of Turbulent Flows with Automatic Change-Point Detection

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Abstract. Research has already shown that turbulent flow consists of some coherent time- and space-organized vortical structures. Some dynamic systems and experimental models are employed to understand the turbulent generation mechanism. However, these approaches still cannot provide a good nonlinear analysis of turbulent time-series. In the real turbulent flow, very complicated nonlinear behaviors, which are affected by many vague factors are present. Based on the nonlinear behavior and the results of from this traditional research, we introduce multivariate statistical analysis of an experimental study to explain practical phenomenon. In this paper, a new approach of fuzzy piecewise regression analysis with automatic change-point detection is proposed to predict the nonlinear time-series of turbulent flows. In order to show the practicality and usefulness of this model, we present an example of predicting the near-wall turbulence time-series as a verifiable model. The results of practical applications show that the proposed method is appropriate and appears to be useful in nonlinear analysis and in fuzzy environments to predict the turbulence time-series.

Key words: change-point, fuzzy regression, near wall turbulent, necessity, possibility, time-series.

1. Introduction

The turbulent flow research has long been dominated by the Reynolds Averaged Navier–Stokes Equations, energy equations and state equations. In recent decades, Direct Numerical Simulation (DNS) and Computational Fluid Dynamics (CFD) code play important roles in turbulent simulation modeling due to the improvement of computer technology.

Most research has focused on understanding the characteristics of turbulence and using semi-empirical theories to fit the experimental data, as evidenced by the volumes of publications involving experimental data, mathematical analysis, and computational modeling, e.g. [1–6]. These methods can be used to estimate the characteristics by using statistical analysis and fitting the parameters from the experimental data.

In near-wall regions, a large production occurs and the presence of nonlinearity becomes significant. Many studies have concentrated on the mechanism of near-wall turbulence and the coherent vortical structure of bursting, e.g. [6, 7]. Aubry et al. [7] employed a dynamical systems approach to study the behavior of streamwise vortices in the near-wall region of turbulent boundary layers. They investigated the intermittent behavior of the streamwise vortices, similar to the bursting events widely observed in the near-wall region and showed that the bursting behavior is produced autonomously near wall and that the structure and duration of the bursts is determined there. They provided a good insight into relating low-dimensional chaotic dynamics to a realistic turbulent boundary layer flow through the formation of longitudinal vortices and the flow evolution based on instability. Hamilton et al. [6] used direct numerical simulations to study the regeneration dynamics of turbulent structure found in the near-wall region. The authors attempted to understand the self-regeneration mechanism and the characteristic spanwise spacing of the structures commonly observed in the near-wall region of turbulent flows. These studies can identify many main features of the near-wall dynamics and provide important information concerning the physical basis of turbulent generation mechanisms. One important feature in the near-wall turbulence is that these instantaneous characteristics of the velocity, intensity and location (relative distance) are all dependent on time-series and are interrelated. Time-series analysis with these correlations then becomes an important clue about how to approach the study of the turbulence. However, in this field only a few papers that appear in publications about turbulent flow depend on time-series [8–12]. Porporato and Ridolfi [12] applied the nonlinear time-series analysis to a near-wall turbulence signal in a hydraulically smooth pipe. They measured the signal using a Laser Doppler Anemometer (LDA) as time-series observation and applied a nonlinear chaotic prediction to a high-dimensional system, thus producing forecasts of rapidly decreasing quality over time. Their work did not tend to practical applications.

In practical circumstances, it is difficult to grasp rules for predicting the nonlinear turbulent behavior and to forecast the velocity and intensities of the turbulent flow at different times. Therefore, the purpose of this paper is to develop a predictable model for grasping the information of the turbulent behaviors in advance and to understand the characteristics by time-series data. Based on this information obtained from the turbulent behaviors, another new fuzzy reasoning rule for prediction on chaotic time-series model could be developed in a subsequent paper.

To accomplish this, we apply a new approach of fuzzy piecewise regression analysis to predict the nonlinear time-series of near-wall turbulent flows with automatic change-point detection. The idea of this method was developed by Tzeng and computer runs were tested in [13, 14]. The method can grasp the dynamics of nonlinear time-series of flow character and path. The observed information of turbulent flow can be reconstructed piecewisely. In order to show the practicality and usefulness of this model, an example for predicting the near-wall turbulence time-series is validated by subjectively taking 11 sample points of the time-series experimental data. From the results of practical applications, the proposed method is appropriate and can be used for nonlinear behavior in fuzzy environments to predict the turbulence time-series.

This paper is organized as follows. Section 2 introduces the idea of developing a new approach for solving nonlinear time-series in near-wall turbulence. Section 3 reviews the conventional fuzzy regression analysis and proposes a fuzzy piecewise regression analysis with automatic change-point detection for predicting turbulent time-series. Section 4 provides an example for predicting the near-wall turbulence time-series with the fuzzy piecewise regression method and discusses the predicted results. Finally, Section 5 presents the conclusions.

2. Developing a New Approach for Solving Nonlinear Time-series in Near-Wall Turbulence

The possibility theory on possibility distribution has been proposed by Zadeh [15] and advanced by Dubois and Prade [16]. Tanaka et al. [17] introduced a linear programming (LP) based regression method using a linear model with symmetrical triangular fuzzy parameters and then defined the possibility and necessity regression analyses [18]. Sakawa and Yano [19, 20] recently generalized the minimization, maximization and conjunction formulation developed by Tanaka and co-workers [18, 21]. However, two weaknesses involving the fuzzy regression model have arisen. First, in possibility analysis, Redden and Woodall [22] demonstrated that Tanaka's methodologies were extremely sensitive to outliers. Furthermore, the fuzzy predictive interval tends to become fuzzier as more data are collected and has no operational definition or interpretation. Second, in necessity analysis, the necessity area would not be obtained owing to the large variation in data [13, 14, 18, 23]. Tanaka et al. [18] suggested a polynomial or nonlinear model to deal with the above problems. Since the distribution of data is probably segmented, Yu et al. [13, 14] proposed two approaches applicable to avoid these problems: one is to use the piecewise model to address the necessity problem and the other is to use the fuzzy piecewise regression to address the automatic change-point detection of nonlinear observations. They proposed a general piecewise necessity regression analysis based on linear programming (LP) to obtain the necessity area from nonlinear data behavior. The unique characteristic of their method is that, according to data distribution, even if the data are irregular, practitioners must specify the number and the positions of change-points [11]. However, as the sample size increases, the number of change-points increases and the piecewise linear interval model also becomes complex. Therefore, controlling the number of change-intervals and obtaining a parsimonious regression model efficiently is a serious problem. Porporato and Ridolfi [12] applied the nonlinear chaotic analysis to a near-wall turbulence signal, measured using a Laser Doppler Anemometer in time-series data. They showed that a nonlinear prediction to a system of high-dimension could produce forecasts of rapidly decreasing quality over time and that it was not suitable for practical applications. Hence, we try to adopt the piecewise concept to implement fuzzy piecewise regression for solving nonlinear turbulent time-series trends. We apply a general fuzzy piecewise regression analysis with automatic change-point detection proposed by Yu et al. [14] to obtain the regression model and change-points simultaneously.

Fuzzy piecewise possibility and necessity regression models are used when the function behaves differently in different parts of the range of crisp input variables. This means that the above problem can be formulated as a mixed-integer programming problem for solving fuzzy piecewise regression. The proposed fuzzy piecewise regression method for solving nonlinear turbulent time-series has four merits: (a) it can be used to predict the nonlinear turbulent time-series data; (b) if the number of change-points is previously specified, then the positions of change-points and the fuzzy piecewise regression model are obtained simultaneously; (c) by using the mixed-integer programming, the solution is the global optimal rather than local optimal solution; (d) it is more robust than conventional fuzzy regression. The conventional regression is sensitive to outliers. In contrast, based on a piecewise concept, the proposed method can deal with outliers by segmenting the data automatically. Therefore, in this work we focus on building an appropriate model that can be easily used to predict the nonlinear trends of turbulence time-series. The details of general fuzzy piecewise regression analysis with automatic change-points detection are provided in Yu et al. [14].

3. Fuzzy Piecewise Regression Model with Automatic Change-Point Detection

In this section, we will build a fuzzy piecewise regression model with automatic change-point detection. The concept of interval arithmetic is introduced in Section 3.1. A fuzzy piecewise regression model with automatic change-point detection applied to predict the nonlinear time-series of turbulent flows is described in Section 3.2.

3.1. THE CONCEPT OF INTERVAL ARITHMETIC

A linear interval model with independent variables is presented using interval parameters A_i $(i = 0, 1, 2, \dots, q)$ as

$$Y(\mathbf{x}_t) = A_0 + A_1 x_{1t} + \cdots + A_q x_{qt}, (1) \tag{1}$$

where $Y(\mathbf{x}_t)$ is the predicted interval corresponding to the input vector $\mathbf{x}_t$ and t is the time datum $(t = 1, 2, \dots, n)$ and $\mathbf{x}_t = (x_{1t}, x_{2t}, \dots, x_{qt})$. In short, $\mathbf{x} = (x_1, x_2, \dots, x_q)$ is a q-dimensional input vector. Throughout this work, closed intervals are denoted by capital letter A and B. An interval is defined by an ordered pair in brackets as

$$A = [a_L, a_R] = [a : a_L \leq a \leq a_R], \tag{2}$$

where a_L denotes the left limit and a_R denotes the right limit of A. Interval A is also denoted by its center and width (radius) as

$$A = (a_c, a_w) = \{a : a_c - a_w \leq a \leq a_c + a_w\}, \tag{3}$$

where a_c denotes the center and a_w denotes the width (radius, $a_w \geq 0$), i.e., half of the width of A. From Equations (2) and (3), the center and the radius of interval A can be calculated as

$$a_c = (a_R + a_L)/2, \tag{4}$$

$$a_w = (a_r - a_L)/2. \tag{5}$$

The following additions and multiplications are used hereafter

$$A + B = (a_c, a_w) + (b_c, b_w) = (a_c + b_c, a_w + b_w), \tag{6}$$

$$rA = r(a_c, a_w) = (ra_c, |r|a_w), \tag{7}$$

where r is the real number.

3.1.1. *Linear Interval Model*

The following linear model of Equation (1) is represented in detail

$$\begin{aligned} Y(\mathbf{x}_t) &= A_0 + A_1 x_{1t} + \cdots + A_q x_{qt} \\ &= (a_{0c}, a_{0w}) + (a_{1c}, a_{1w}) x_{1t} + \cdots + (a_{qc}, a_{qw}) x_{qt} \\ &= (Y_c(\mathbf{x}_t), Y_w(\mathbf{x}_t)), \end{aligned} \tag{8}$$

$$Y_c(\mathbf{x}_t) = a_{0c} + a_{1c} x_{1t} + \cdots + a_{qc} x_{qt}, \tag{9}$$

$$Y_w(\mathbf{x}_t) = a_{0w} + a_{1w}|x_{1t}| + \cdots + a_{qw}|x_{qt}|, \tag{10}$$

where $Y_c(\mathbf{x}_t)$ represents the center and $Y_w(\mathbf{x}_t)$ is the width of the predicted interval $Y(\mathbf{x}_t)$.

3.1.2. *Possibility Regression Analysis*

$$\begin{aligned} Y^*(\mathbf{x}_t) &= A_0^* + A_1^* x_{1t} + \cdots + A_q^* x_{qt} \\ &= (a_{0c}^*, a_{0w}^*) + (a_{1c}^*, a_{1w}^*) x_{1t} + \cdots + (a_{qc}^*, a_{qw}^*) x_{qt} \\ &= (Y_c^*(\mathbf{x}_t), Y_w^*(\mathbf{x}_t)), \end{aligned} \tag{11}$$

which satisfies the following conditions:

$$Y_t \subseteq Y^*(\mathbf{x}_t), \quad t = 1, 2, \ldots, n, \tag{12}$$

where Y_t is the tth observation time.

3.1.3. *Minimization Problem for Interval-Valued Data*

$$\text{Minimize} \quad Y_w^*(\mathbf{x}_1) + Y_w^*(\mathbf{x}_2) + \cdots + Y_w^*(\mathbf{x}_n) \tag{13}$$

$$\text{Subject to} \quad Y^*(\mathbf{x}_t) \supseteq Y_t, \quad t = 1, 2, \ldots, n \tag{14}$$

$$a_{iw}^* \geq 0, \quad i = 1, 2, \ldots, q, \tag{15}$$

This LP problem is written as follows:

$$\text{Minimize} \quad \sum_{t=1}^{n} (a_{0w}^* + a_{1w}^* |x_{1t}| + \cdots + a_{qw}^* |x_{qt}|) \tag{16}$$

$$\text{Subject to} \quad \left(a_{0c}^* + \sum_{i=1}^{q} a_{ic}^* x_{it}\right) - \left(a_{0w}^* + \sum_{i=1}^{q} a_{iw}^* |x_{it}|\right) \leq Y_{tL}, \quad t = 1, 2, \ldots, n \tag{17}$$

$$\left(a_{0c}^* + \sum_{i=1}^{q} a_{ic}^* x_{it}\right) + \left(a_{0w}^* + \sum_{i=1}^{q} a_{iw}^* |x_{it}|\right) \geq Y_{tR}, \quad t = 1, 2, \ldots, n \tag{18}$$

$$a_{0w}^*, a_{iw}^* \geq 0, \quad i = 1, 2, \ldots, q. \tag{19}$$

The weakness in the above model is that it is sensitive to outliers. The model used to have larger possibility than the system should have and used to be warped and bent too much by various fluctuating data. The fuzzy predictive model tends to become fuzzier as more data are collected and has no operational definition or interpretation.

3.1.4. *Necessity Regression Analysis*

$$\begin{aligned} Y_*(\mathbf{x}_t) &= A_{0*} + A_{1*} x_{1t} + \cdots + A_{q*} x_{qt} \\ &= (a_{0c^*}, a_{0w^*}) + (a_{1c^*}, a_{1w^*}) x_{1t} + \cdots + (a_{qc^*}, a_{qw^*}) x_{qt} \\ &= (Y_{c^*}(\mathbf{x}_t), Y_{w^*}(\mathbf{x}_t)), \end{aligned} \tag{20}$$

which satisfies the following conditions:

$$Y_*(\mathbf{x}_t) \subseteq Y_t, \quad t = 1, 2, \ldots, n. \tag{21}$$

3.1.5. *Maximization Problem for Interval-Valued Data*

$$\text{Maximize} \quad Y_{w^*}(\mathbf{x}_1) + Y_{w^*}(\mathbf{x}_2) + \cdots + Y_{w^*}(\mathbf{x}_n) \tag{22}$$

$$\text{Subject to} \quad Y_*(\mathbf{x}_t) \subseteq Y_t, \quad t = 1, 2, \ldots, n \tag{23}$$

$$a_{iw^*} \geq 0, \quad i = 1, 2, \ldots, q. \tag{24}$$

This LP problem is written as follows:

$$\text{Maximize} \quad \sum_{t=1}^{n} (a_{0w^*} + a_{1w^*}|x_{1t}| + \cdots + a_{qw^*}|x_{qt}|) \tag{25}$$

$$\text{Subject to} \quad \left(a_{0c^*} + \sum_{i=1}^{q} a_{ic^*} x_{it}\right) - \left(a_{0w^*} + \sum_{i=1}^{q} a_{iw^*}|x_{it}|\right) \geq Y_{tL},$$

$$t = 1, 2, \ldots, n \tag{26}$$

$$\left(a_{0c^*} + \sum_{i=1}^{q} a_{ic^*} x_{it}\right) + \left(a_{0w^*} + \sum_{i=1}^{q} a_{iw^*}|x_{it}|\right) \leq Y_{tR},$$

$$t = 1, 2, \ldots, n \tag{27}$$

$$a_{0w^*}, a_{iw^*} \geq 0, \quad i = 1, 2, \ldots, q. \tag{28}$$

The above LP formulation of necessity has no feasible solution owing to large fluctuations of the given data. Therefore, a fuzzy piecewise regression was proposed by [13] for treating such problem. In [13], the change-points are given before employing the above method. For overcoming this restriction, Yu et al. [14] developed a general fuzzy piecewise regression analysis with automatic change-point detection.

3.2. FUZZY PIECEWISE REGRESSION MODEL WITH AUTOMATIC CHANGE-POINT DETECTION FOR PREDICTING THE NONLINEARITY OF TURBULENCE TIME-SERIES

Change-points, which are the joints of pieces, are quoted from conventional statistical piecewise regression [24]. This terminology is used through out the research. For an input single variable x, the time-serial points $\{p_1, p_2, \ldots, p_k\}$ are the values of single variable x and are subject to an ordering constraint $p_1 < p_2 < \ldots < p_k$, $k \leq n$. This work of Yu et al. [14] assumes that every datum is a potential change-point except p_k. Therefore, change-point alternatives are in the initial possibility and necessity regression model. The suspected positions of change-points are $\mathbf{p} = \{p_1, p_2, \ldots, p_{k-1}\}$.

DEFINITION 1. The point p_j is a change-point if $B(|b_c| \geq \delta, b_w)$. δ is a small positive value specified by the user. Otherwise, p_j is not a change-point if $B(|b_c| < \delta, b_w = 0)$.

If p_j is a change-point, then the operation of piecewise term is as follows:

$$(|x_t - p_j| + x_t - p_j)/2 = \begin{cases} x_t - p_j, & \text{if } x_t > p_j, \\ 0, & \text{if } x_t \leq p_j, \end{cases} \tag{29}$$

where $j = 1, 2, \ldots, k-1$.

An LP formulation is presented to determine the possibility area and necessity area by the piecewise linear interval model. For the sake of simplicity, we demonstrate the proposed method with the piecewise interval model by using an input single variable x

$$\begin{aligned} \hat{Y}(x_t) &= h(x_t) + B_1(|x_t - p_1| + x_t - p_1)/2 \\ &\quad + B_2(|x_t - p_2| + x_t - p_2)/2 + \cdots \\ &\quad + B_{k-2}(|x_t - p_{k-2}| + x_t - p_{k-2})/2 \\ &\quad + B_{k-1}(|x_t - p_{k-1}| + x_t - p_{k-1})/2, \\ h(x(t)) &= A_0 + A_1 x_t. \end{aligned} \tag{30}$$

Equation (30) represents $Y^*(x_t)$ and $Y_*(x_t)$ in the initial possibility and necessity model respectively. After prespecifying the number of change-points by practitioners, the fuzzy regression model and the positions of the change-points are obtained simultaneously. The piecewise terms of LP formulation are as follows:

$$\begin{aligned} &B_1(|x_t - p_1| + x_t - p_1)/2 + B_2(|x_t - p_2| + x_t - p_2)/2 + \cdots \\ &\quad + B_{k-2}(|x_t - p_{k-2}| + x_t - p_{k-2})/2 \\ &\quad + B_{k-1}(|x_t - p_{k-1}| + x_t - p_{k-1})/2, \quad t = 1, 2, \ldots, n. \end{aligned} \tag{31}$$

The difference between Equations (30) and (11) is Equation (31). That is the initial piecewise expression for the given data.

$$\begin{aligned} &B_1(|x_t - p_1| + x_t - p_1)/2 + B_2(|x_t - p_2| + x_t - p_2)/2 + \cdots \\ &\qquad + B_{k-2}(|x_t - p_{k-2}| + x_t - p_{k-2})/2 \\ &\qquad + B_{k-1}(|x_t - p_{k-1}| + x_t - p_{k-1})/2 \\ &\quad = \sum_{j=1}^{k-1} b_{jc}(|x_t - p_j| + x_t - p_j)/2 \\ &\qquad + \sum_{j=1}^{k-1} b_{jw}(|x_t - p_j| + x_t - p_j)/2, \quad t = 1, 2, \ldots, n. \end{aligned} \tag{32}$$

The initial possibility formulation with q input variables (multiple variables of effect factors) is as follows:

$$Y^*(\mathbf{x}_t) = A_0^* + \sum_{i=1}^{q} A_i^* x_{it} + \sum_{i=1}^{q}\sum_{j=1}^{k_t-1} B_{ij}^*(|x_{it} - p_j| + x_{it} - p_j)/2, \quad t = 1, 2, \ldots, n. \tag{33}$$

3.2.1. *Possibility Analysis with Automatic Change-Point Detection*

Minimize

$$Z_p = \sum_{t=1}^{n}\left[a_{0w}^* + \sum_{i=1}^{q} a_{iw}^* x_{it} + \sum_{i=1}^{q}\sum_{j=1}^{k_t-1} b_{ijw}^*(|x_{it} - p_j| + x_{it} - p_j)/2\right] \tag{34}$$

Subject to

$$\left[a_{0c}^* + \sum_{i=1}^{q} a_{ic}^* x_{it} + \sum_{i=1}^{q}\sum_{j=1}^{k_*-1} b_{ijc}^*(|x_{it} - p_j| + x_{it} - p_j)/2\right] - \left[a_{0w}^* + \sum_{i=1}^{q} a_{iw}^* x_{it} + \sum_{i=1}^{q}\sum_{j=1}^{k_*-1} b_{ijw}^*(|x_{it} - p_j| + x_{it} - p_j)/2\right] \le Y_{tL},$$
$$t = 1, 2, \ldots, n, \tag{35}$$

$$\left[a_{0c}^* + \sum_{i=1}^{q} a_{ic}^* x_{it} + \sum_{i=1}^{q}\sum_{j=1}^{k_*-1} b_{ijc}^*(|x_{it} - p_j| + x_{it} - p_j)/2\right] + \left[a_{0w}^* + \sum_{i=1}^{q} a_{iw}^* x_{it} + \sum_{i=1}^{q}\sum_{j=1}^{k_*-1} b_{ijw}^*(|x_{it} - p_j| + x_{it} - p_j)/2\right] \ge Y_{tL},$$
$$t = 1, 2, \ldots, n, \tag{36}$$

$$b_{ijc}^* < Mu_t - 2\delta v_j + \delta + \varphi v_j,$$
$$i = 1, 2, \ldots, q;\ j = 1, 2, \ldots, k_t - 1;\ t = 1, 2, \ldots, n, \tag{37}$$

$$b_{ijc}^* > 2\delta u_j - Mv_j - \delta - \varphi u_j,$$
$$i = 1, 2, \ldots, q;\ j = 1, 2, \ldots, k_t - 1;\ t = 1, 2, \ldots, n. \tag{38}$$

$$v_j + u_j = I_j, \quad j = 1, 2, \ldots, k_t - 1;\ t = 1, 2, \ldots, n, \tag{39}$$

$$\sum_{j=1}^{m-1} I_j \leq C, \tag{40}$$

$$b^*_{jw} \leq MI_j, \quad j = 1, 2, \ldots, k_t - 1; t = 1, 2, \ldots, n. \tag{41}$$

The initial necessity formulation with input variables (multiple variables) is as follows:

$$Y_*(x_t) = A_{*0} + \sum_{i=1}^{q} A_{i*}x_{it} + \sum_{i=1}^{q}\sum_{j=1}^{k_i-1} B_{ij*}(|x_{it} - p_j| + x_{it} - p_j)/2, \quad t = 1, 2, \ldots, n. \tag{42}$$

3.2.2. *Necessity Analysis with Automatic Change-Point Detection*

Maximize

$$Z_p = \sum_{t=1}^{n}\left[a_{0w*} + \sum_{i=1}^{q} a_{iw*}x_{it} + \sum_{i=1}^{q}\sum_{j=1}^{k_t-1} b_{ijw*}(|x_{it} - p_j| + x_{it} - p_j)/2\right] \tag{43}$$

Subject to

$$\left[a_{0c*} + \sum_{i=1}^{q} a_{ic*}x_{it} + \sum_{i=1}^{q}\sum_{j=1}^{k_*-1} b_{ijc*}(|x_{it} - p_j| + x_{it} - p_j)/2\right] - \left[a_{0w*} + \sum_{i=1}^{q} a_{iw*}x_{it} + \sum_{i=1}^{q}\sum_{j=1}^{k_*-1} b_{ijw*}(|x_{it} - p_j| + x_{it} - p_j)/2\right] \leq Y_{tL},$$
$$t = 1, 2, \ldots, n, \tag{44}$$

$$\left[a_{0c*} + \sum_{i=1}^{q} a_{ic*}x_{it} + \sum_{i=1}^{q}\sum_{j=1}^{k_*-1} b_{ijc*}(|x_{it} - p_j| + x_{it} - p_j)/2\right] + \left[a^*_{0w} + \sum_{i=1}^{q} a^*_{iw}x_{it} + \sum_{i=1}^{q}\sum_{j=1}^{k_*-1} b^*_{ijw}(|x_{it} - p_j| + x_{it} - p_j)/2\right] \geq Y_{tL},$$
$$t = 1, 2, \ldots, n, \tag{45}$$

$$b_{ijc*} < Mu_t - 2\delta v_j + \delta + \varphi v_j,$$
$$i = 1, 2, \ldots, q; j = 1, 2, \ldots, k_t - 1; t = 1, 2, \ldots, n, \tag{46}$$

$$b_{ijc*} > 2\delta u_j - Mv_j - \delta - \varphi u_j,$$
$$i = 1, 2, \ldots, q;\ j = 1, 2, \ldots, k_t - 1;\ t = 1, 2, \ldots, n. \tag{47}$$

$$v_j + u_j = I_j, \quad j = 1, 2, \ldots, k_t - 1;\ t = 1, 2, \ldots, n, \tag{48}$$

$$\sum_{j=1}^{m-1} I_j \leq C, \tag{49}$$

$$b_{jw*} \leq MI_j, \quad j = 1, 2, \ldots, k_t - 1;\ t = 1, 2, \ldots, n. \tag{50}$$

where $u_j, v_j, I_j \in \{0, 1\}$; δ denotes a small positive value and $\delta > \varphi$; b^*_{ijw} and b_{ijw*} are unconstrained in sign. M is a large positive constant and can be specified as $M \geq \text{Max}\{|b^*_{ijc}|$ and $|b_{ijc*}|, i = 1, 2, \ldots, q, j = 1, 2, \ldots, k_t - 1$ and $t = 1, 2, \ldots, n\}$. Equations (37) to (41) and Equations (46) to (50) are checked to see whether they satisfy the definition of change-point in Definition 1. For example, in possibility regression analysis:

(i) If $b^*_{ijw} \geq \delta$, then $u_j = 1$ and $v_j = 0$ (from Equations (37) to (39));
(ii) If $b^*_{ijw} \leq -\delta$, then $u_j = 0$ and $v_j = 1$ (from Equations (37) to (39));
(iii) If $|b^*_{ijc}| < \delta$, then $u_j = v_j = 0$ (from Equations (37) to (39));
we know (i), (ii) and (iii) demonstrate that if $|b^*_{ijc}| \geq \delta$, (i.e., a change-point occurs in the jth position), then $u_j + v_j = I_j = 1$, and otherwise $u_j + v_j = I_j = 0$;
(iv) $\sum_{j=1}^{m-1} I_J \leq C$ limits the number of change-points. C is a reasonable measurement of the number of change-points that depend on data distribution (from Equation (40));
(v) $b_{jw*} \leq MI_j$ implies if $b^*_{ijc} = 0$ then $b^*_{ijw} = 0$ (from 41).

Practitioners previously specified the number of change points (C) by using observed data (plotted drawing). Then, they can obtain the positions of change interval and the fuzzy piecewise regression model simultaneously. In possibility analysis, based on piecewise characteristics, our methodology is insensitive to outliers. Furthermore, the fuzzy predicted interval does not tend to become fuzzier as more data are collected. In necessity analysis, practitioners do not need to consider the cases in which we could not obtain a necessity area regardless of regular or irregular data. All the practitioners have to do is to specify the conceivable number of change-points (C). The detected change-points or their neighborhood can be viewed as outliers under some circumstances. The proposed piecewise method is more robust than the conventional fuzzy regression. By using mixed integer programming, this method can give the global optimal solution to grasp the trends for predicting the nonlinearity of turbulence time-series.

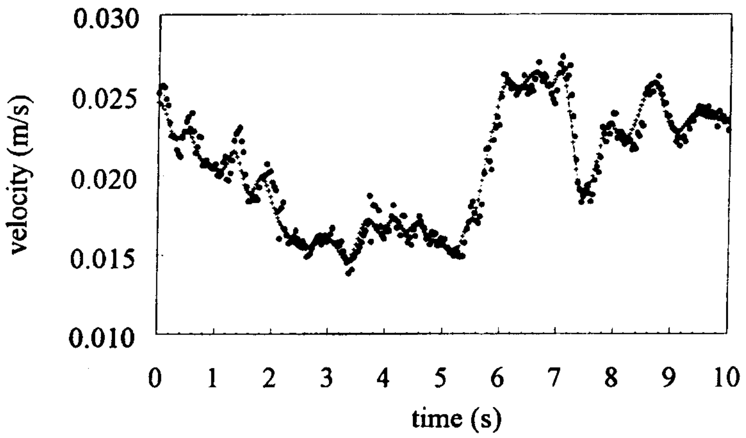

Figure 1. The relation between times (s) and velocity (m/s) by time-series experimental data (from Porporato and Ridolfi, [12]).

4. Case of Nonlinear Type of Near-Wall Turbulence Time-Series

In this section, in order to show the practicality and usefulness of the method described in the above section, a case for predicting the near-wall turbulence time-series is taken as a validated model. Porporato and Ridolfi [12] discovered a phenomenon which the near-wall turbulence time-serial data is nonlinear from the experiments, measured using a Laser Doppler Anemometer. They applied a nonlinear chaotic prediction to a high-dimension system and produced forecasts of rapidly decreasing quality over time, with no consequences for practical applications. They use a trace of the direct prediction with forecast interval $= 5 \times 3 \times \Delta t =$ 160.40 ms: the correspondence with reality is optimum and maintained during the strong and extended velocity gradients. Their result is shown in Figure 1. Although the forecast is worse globally and the rapid oscillations largely escape the method, the large-scale behavior is still well captured, and even the strong velocity gradients are forecasted with an accuracy equal to that of the small- scale motions by their research. However, when forecast interval is slightly above the traditional Kolmogorov time-scale, the forecast is rather poor.

Therefore, we try to use a new approach of fuzzy piecewise regression analysis to predict the nonlinear time-series of turbulent flows with automatic change-point detection. The idea of this method was developed by Tzeng and computer runs were tested by Yu [13, 14]. The testing data used here as an example were taken subjectively using eleven equally spaced sample points of time-series from Figure 1. By using the piecewise concept, the possibility and necessity area represented by a piecewise linear interval model rather than a nonlinear interval regression one can be obtained. By adjusting the under terms of $h(x)$, the proposed method can also be presented as the nonlinear interval model in time period of time-series. In the following example, for the sake of simplicity, $h(x)$ is a linear interval function. If the property of data is nonlinear as Figure 1 (near-wall turbulence time-series), we can adjust the order term of $h(x)$ and, then, $Y^*(x)$ and $Y_*(x)$ will turn into a nonlinear piecewise interval regression model.

4.1. Testing model and explanation

EXAMPLE. Let us consider the following interval of data value according to Figure 1 to test and explain this concept easily, and assume taking rough raw data as follows:

$$\begin{aligned}\{(x, y)\} = \{&(0, [0.0248, 0.0260]), (1, [0.0203, 0.0210]), (2, [0.0185, 0.0208]),\\ &(3, 0.0161, 0.0173]), (4, [0.0167, 0.0189]), (5, [0.0160, 0.0165]),\\ &(6, [0.0235, 0.0270]), (7, [0.0270, 0.0282]), (8, [0.0227, 0.0245]),\\ &(9, [0.0225, 0.0238]), (10, [0.0232, 0.0243]).\end{aligned}$$

We divided the time-serial data into 10 segments from the experimental results and use 11 samples of time-series to test this method. Very few data is used in order to demonstrate the practicality of this model. The time-series x_t ranges from 0 to 10. We use $p_1 = 0$, $p_2 = 1$, $p_3 = 2$, $p_4 = 3$, $p_5 = 4$, $p_6 = 5$, $p_7 = 6$, $p_8 = 7$, $p_9 = 8$, $p_{10} = 9$, $p_{11} = 10$ as change-points.

The initial possibility model of nonlinear time-series shown as linear piecewise model is as follows:

$$\begin{aligned}Y^*(x_t) = {}&(a^*_{0c}, a^*_{0w}) + (a^*_{1c}, a^*_{1w})x_t + (b^*_{1c}, b^*_{1w})(|x_t - 0| + x_t - 0)/2\\ &+ (b^*_{2c}, b^*_{2w})(|x_t - 1| + x_t - 1)/2 + (b^*_{3c}, b^*_{3w})(|x_t - 2| + x_t - 2)/2\\ &+ (b^*_{4c}, b^*_{4w})(|x_t - 3| + x_t - 3)/2 + (b^*_{5c}, b^*_{5w})(|x_t - 4| + x_t - 4)/2\\ &+ (b^*_{6c}, b^*_{6w})(|x_t - 5| + x_t - 5)/2 + (b^*_{7c}, b^*_{7w})(|x_t - 6| + x_t - 6)/2\\ &+ (b^*_{8c}, b^*_{8w})(|x_t - 7| + x_t - 7)/2 + (b^*_{9c}, b^*_{9w})(|x_t - 8| + x_t - 8)/2\\ &+ (b^*_{10c}, b^*_{10w})(|x_t - 9| + x_t - 9)/2\\ &+ (b^*_{11c}, b^*_{11w})(|x_t - 10| + x_t - 10)/2.\end{aligned}$$

The initial necessity model of nonlinear time-series shown as linear piecewise model is as follows:

$$\begin{aligned}Y_*(x_t) = {}&(a_{0c*}, a_{0w*}) + (a_{1c*}, a_{1w*})x_t + (b_{1c*}, b_{1w*})(|x_t - 0| + x_t - 0)/2\\ &+ (b_{2c*}, b_{2w*})(|x_t - 1| + x_t - 1)/2\\ &+ (b_{3c*}, b_{3w*})(|x_t - 2| + x_t - 2)/2\\ &+ (b_{4c*}, b_{4w*})(|x_t - 3| + x_t - 3)/2\end{aligned}$$

$$
\begin{aligned}
&+ (b_{5c*}, b_{5w*})(|x_t - 4| + x_t - 4)/2 \\
&+ (b_{6c*}, b_{6w*})(|x_t - 5| + x_t - 5)/2 \\
&+ (b_{7c*}, b_{7w*})(|x_t - 6| + x_t - 6)/2 \\
&+ (b_{8c*}, b_{8w*})(|x_t - 7| + x_t - 7)/2 \\
&+ (b_{9c*}, b_{9w*})(|x_t - 8| + x_t - 8)/2 \\
&+ (b_{10c*}, b_{10w*})(|x_t - 9| + x_t - 9)/2 \\
&+ (b_{11c*}, b_{11w*})(|x_t - 10| + x_t - 10)/2.
\end{aligned}
$$

Let $M = 1000$, $\delta = 0.0000001$, $\varphi = 0.00000001$, $C = 7$. Solving this program by LINDO, we also try to test the case of nonlinear time-series shown as a quadratic piecewise model.

4.2. Results and Discussions

4.2.1. *Results*

Depicts the running results of the possibility and necessity analysis. The following piecewise models are subsequently obtained.

(a) *Nonlinear time-series shown as linear piecewise model*

(i) Possibility

$$
\begin{aligned}
Y^*(x_t) = {} & 0.02445 - 0.0026x_t + 0.001525(|x_t - 3| + x_t - 3) \\
& + 0.0039(|x_t - 5| + x_t - 5) - 0.0042(|x_t - 6| + x_t - 6) \\
& \pm [0.00155 + 0.00025(|x_t - 3| + x_t - 3)]
\end{aligned}
$$

(average width: 0.002186 m/s; i.e., total vagueness is 0.01881 m/s)

(ii) Necessity

$$
\begin{aligned}
Y_*(x_t) = {} & 0.0259 - 0.0055x_t + 0.00195(|x_t - 1| + x_t - 1) \\
& + 0.0006(|x_t - 3| + x_t - 3) + 0.005409375(|x_t - 5| + x_t - 5) \\
& - 0.0049875(|x_t - 6| + x_t - 6) \\
& - 0.00233125(|x_t - 7| + x_t - 7) + 0.002275(|x_t - 8| + x_t - 8) \\
& \pm [0.0001 + 0.00008125(|x_t - 5| + x_t - 5)]
\end{aligned}
$$

(average width: 0.000211 m/s; i.e., total vagueness is 0.00231875 m/s)

Table I. The comparison of observed output with predicted output by linear piecewise model.

Sample (time, s)	observed output (raw data, m/s)	Predicted output (m/s) possibility (3 change-points)	necessity (6 change-points)
0	[0.0248, 0.0260]	[0.02290, 0.02600]	[0.02580, 0.02600]
1	[0.0203, 0.0210]	[0.02030, 0.03400]	[0.02030, 0.02050]
2	[0.0185, 0.0208]	[0.01770, 0.02080]	[0.01870, 0.01890]
3	[0.0161, 0.0173]	[0.01510, 0.01820]	[0.01710, 0.01730]
4	[0.0167, 0.0189]	[0.01505, 0.01915]	[0.01670, 0.01690]
5	[0.0160, 0.0165]	[0.01500, 0.02010]	[0.01630, 0.01650]
6	[0.0235, 0.0270]	[0.02275, 0.02885]	[0.02664, 0.02700]
7	[0.0270, 0.0282]	[0.02210, 0.02920]	[0.02700, 0.02753]
8	[0.0227, 0.0245]	[0.02145, 0.02955]	[0.02270, 0.02339]
9	[0.0225, 0.0238]	[0.02080, 0.02990]	[0.02295, 0.02380]
10	[0.0232, 0.0243]	[0.02015, 0.03025]	[0.02320, 0.02421]

which all satisfies the conditions:

$$Y_*(x_t) \subseteq Y_t \subseteq Y^*(x_t), \quad t = 0, 1, 2, \ldots, 10.$$

[necessity width: 0.000211 m/s] $\leq$ [raw data (observation) width: 0.001545 m/s] $\leq$ [possibility width: 0.002186 m/s]. Similarity index = 0.71 (raw data width/possibility width).

The comparison of observed output with predicted output by the linear piecewise model is shown as Table I, Figures 2 and 3. The linear piecewise model detects three change-points in possibility analysis and six change-points in necessity analysis.

(b) *Nonlinear time-series shown as quadratic piecewise model*

(i) Possibility

$$\begin{aligned} Y^*(x_t) &= 0.025110 - 0.003930x_t + 0.000490x_t^2 \\ &\quad + 0.003040(|x_t - 5| + x_t - 5) - 0.003680(|x_t - 6| + x_t - 6) \\ &\quad - 0.00223(|x_t - 7| + x_t - 7) \pm [0.001510 + 0.000040x_t] \end{aligned}$$

(average width: 0.001710 m/s; i.e., total vagueness is 0.01881 m/s)

(ii) Necessity

$$\begin{aligned} Y_*(x_t) &= 0.025050 - 0.005450x_t + 0.001150x_t^2 \\ &\quad - 0.001175(|x_t - 2| + x_t - 2) - 0.001625(|x_t - 4| + x_t - 4) \end{aligned}$$

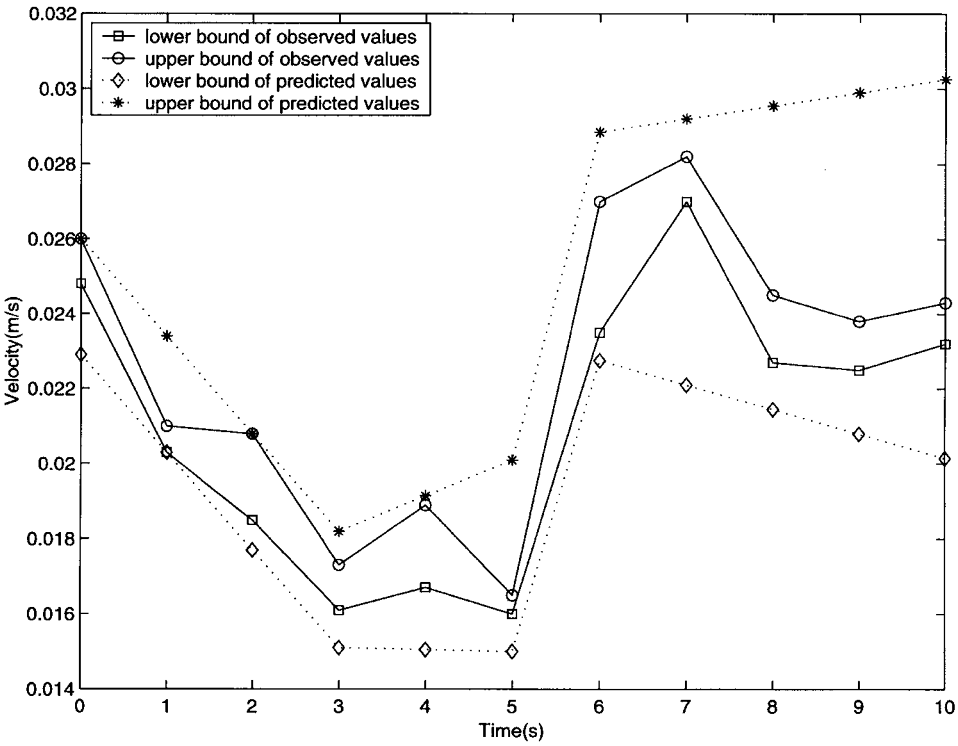

Figure 2. Possibility analysis compared with raw data by linear piecewise model (with 3 change-points: $t = 3, 5, 6$).

$$+ 0.004434(|x_t - 5| + x_t - 5) - 0.005875(|x_t - 6| + x_t - 6)$$
$$- 0.003450(|x_t - 7| + x_t - 7) \pm 0.00025$$

(average width: 0.000250 m/s; i.e., total vagueness is 0.001684375 m/s)

which all satisfies the conditions:

$$Y_*(x_t) \subseteq Y_t \subseteq Y^*(x_t), \quad t = 0, 1, 2, \ldots, 10.$$

[necessity width: 0.000250 m/s] $\leq$ [raw data (observation) width: 0.001545 m/s] $\leq$ [possibility width: 0.001710 m/s]. Similarity index = 0.90 (raw data width/possibility width).

The comparison of observed output with predicted output by the quadratic piecewise model is shown as Table II, Figures 4 and 5. The quadratic piecewise model detects three change-points in possibility analysis and five change-points in necessity analysis.

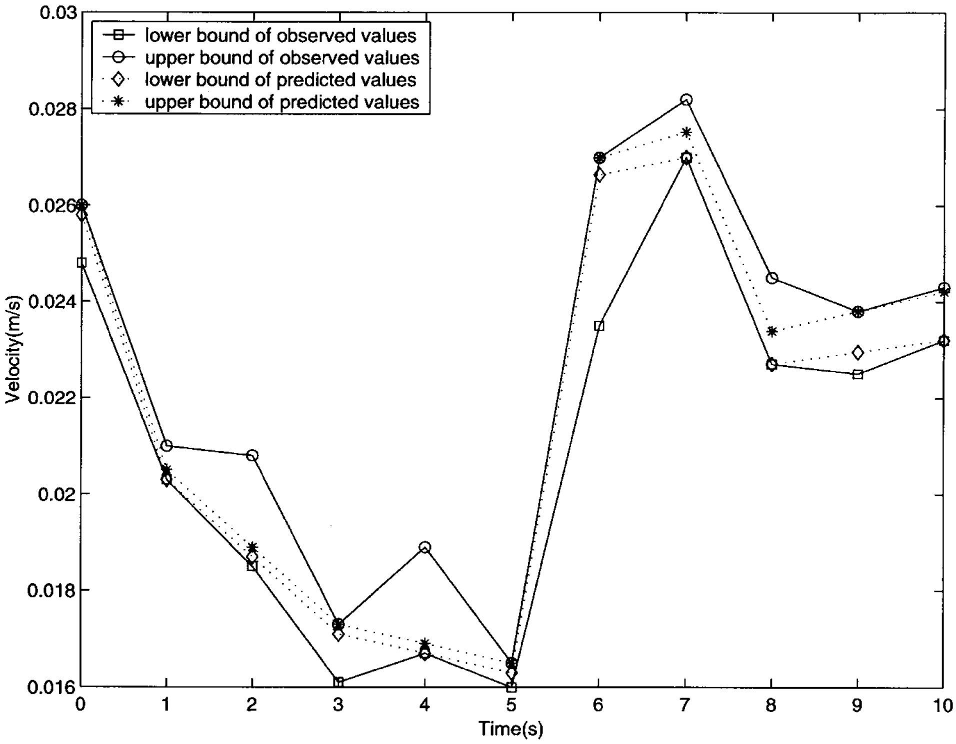

Figure 3. Necessity analysis compared with raw data by linear piecewise model (with 6 change-points: $t = 1, 3, 5, 6, 7, 8$).

Table II. The comparison of observed output with predicted output by quadratic piecewise model.

Sample (time, s)	observed output (raw data, m/s)	Predicted output (m/s) possibility (3 change-points)	necessity (5 change-points)
0	[0.0248, 0.0260]	[0.02360, 0.02662]	[0.02480, 0.02530]
1	[0.0203, 0.0210]	[0.02012, 0.02322]	[0.02050, 0.02100]
2	[0.0185, 0.0208]	[0.01762, 0.02080]	[0.01850, 0.01900]
3	[0.0161, 0.0173]	[0.01610, 0.01936]	[0.01645, 0.01695]
4	[0.0167, 0.0189]	[0.01556, 0.01890]	[0.01670, 0.01720]
5	[0.0160, 0.0165]	[0.01600, 0.01942]	[0.01600, 0.01650]
6	[0.0235, 0.0270]	[0.02350, 0.02700]	[0.02647, 0.02697]
7	[0.0270, 0.0282]	[0.02462, 0.02820]	[0.02749, 0.02799]
8	[0.0227, 0.0245]	[0.02226, 0.02592]	[0.02390, 0.02440]
9	[0.0225, 0.0238]	[0.02088, 0.02462]	[0.02262, 0.02312]
10	[0.0232, 0.0243]	[0.02048, 0.02430]	[0.02364, 0.02414]

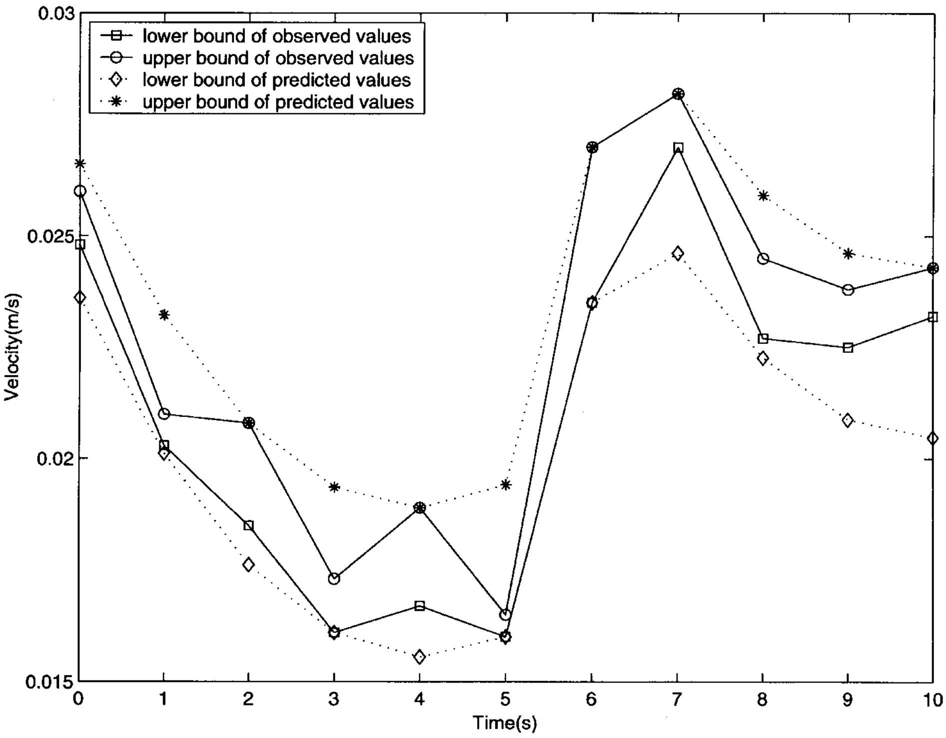

Figure 4. Possibility analysis compared with raw data by quadratic piecewise model (with 3 change-points: $t = 5, 6, 7$).

4.2.2. *Discussions*

Figures 2 and 3 depict the change intervals of the possibility and necessity area for linear piecewise model. According to Table I, Figures 2 and 3, both of the change points in the time interval of the possibility and necessity models appear at the same positions, time $t = 3, 5, 6$ s. During the time period 0 to 3 s and 6 to 10 s, the necessity analysis increases three change-points, at time $t = 1, 7, 8$ s respectively, and these change-point intervals in the time-slice are included in the observed data and possibility distribution. In the mean time, Figures 4 and 5 depict the change intervals of the possibility and necessity area for quadratic piecewise model. According to Table II, Figures 4 and 5, we also find the change points in the time interval of the possibility and necessity models appear at the same positions, time $t = 5, 6, 7$ s. During the time interval, the velocity fluctuation of bursting is predicted from the change-points automatically. In the time period 0 to 5 s, the necessity increases two change-points, at time $t = 2$ and 4 s, and these two change-point intervals in the time-slice are also included in the observed data and possibility distribution.

Based on the results of objective value (total/average vagueness) and similarity index from Tables I and II and Figures 2–5, we can judge that the nonlinear

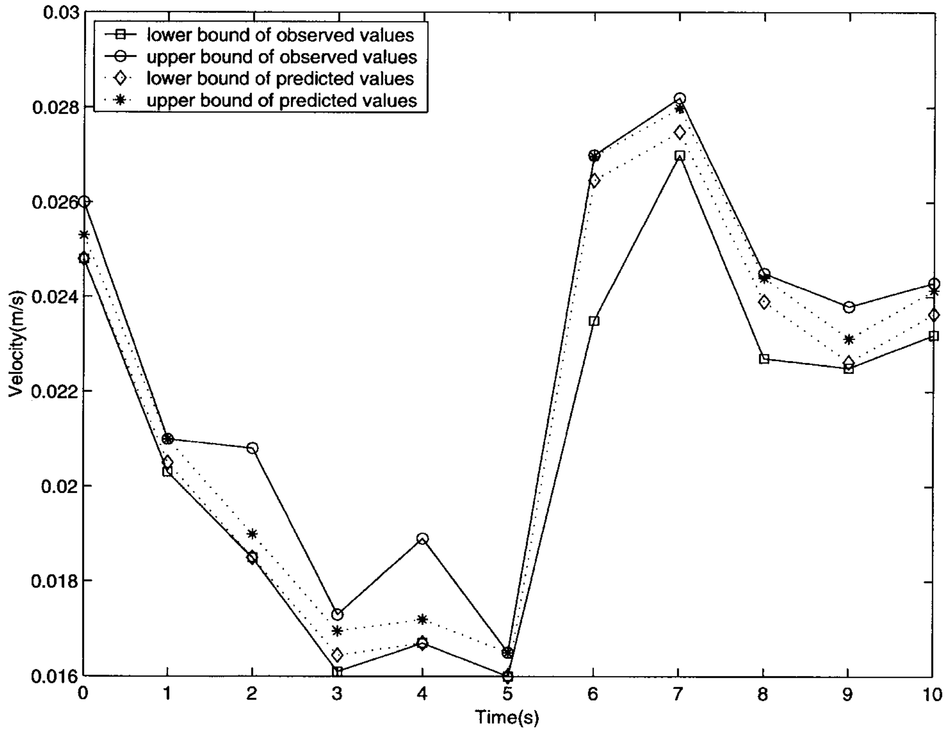

Figure 5. Necessity analysis compared with raw data by quadratic piecewise model (with 5 change-points: $t = 2, 4, 5, 6, 7$).

piecewise model is better than the linear piecewise model (closer observation) in possibility and necessity analysis. In this model the concept of each assuming time point is a change-point except the last one is interpolation. However, C in Equation (49) controls the number of change-points to reduce the redundant change-points. Hence, in order to obtain an effective and parsimonious form, deciding the number of change-points must depend on the observed data distribution and the number of change-points is best minimized [13].

In the present validated example, the fuzzy piecewise regression model captures the regeneration of bursting cycle using the automatic change-point detection to obtain the positions of change-points, at the same time steps $t = 5, 6, 7$ s in both possibility and necessity models, and it predicts the velocity fluctuation for a short time period. The model extracts the typical velocity oscillations during bursting. The regeneration of bursting cycle can be reconstructed and identified from the possibility and necessity analysis of the time-series of velocity component in the model (Figures 2–5). It also mimics a bursting cycle in a short time period successfully. The strategy can be applied for long-term time-series prediction of turbulence by extending the flow sampling in higher dimension with multivariate model. The fuzzy piecewise multivariate-regression model then can be obtained.

The system is complex and is beyond the scope of the present study. Interesting readers may refer to [14] for the extension. It has already been shown that the analysis can obtain global optimal rather than local optimal solution by employing the mixed integer programming [14]. The nonlinear prediction of chaotic dynamics in [12] provided better forecasts with local-type methods than global method. Their restrictions are due to the constraints in the reconstruction of the complex high-dimension system from only one velocity component measurement. However, the fuzzy piecewise analysis is free from the restriction. It permits multivariate data with high dimensional system and detects the change-points automatically [13]. A multivariate example with higher dimension is demonstrated in [14].

According to the prediction results in the verified example, the practicality of the fuzzy piecewise regression model is shown to contain the characteristics and the trends of the relationship between velocity-change and time-series appearing in the nonlinear types of near-wall turbulence. The proposed method, especially the quadratic piecewise model, is appropriate and appears to be useful in searching the change-points of nonlinear near-wall turbulence flow for predicting the turbulence time-series automatically as well as other fuzzy environment. When increasing the sample points, we can improve the prediction result. By decreasing the interval of the possibility area and increasing the interval of the necessity area, a reconstruction of the whole dynamic system is possible. However, as the sample size increases, the number of change-points increases and the interval model become complex. The number of change-points cannot be best minimized easily [14].

In the real world, the environment is uncertain and we must use little data through a sensor or monitoring detection to predict situations in a short time. Also, we must preprocess the data. Time-series analysis uses only the time-series of observations of the variable being forecasted in order to develop a model for predicting future values. A broad classification of some major forecasting methods is shown in the Appendix [25]. The time-series methods can be categorized into several classes, including the traditional time-series methods, the Box–Jenkins model, the dynamic models (i.e., Bayesian theory, Kalman filtering, chaos, etc.), the fuzzy time-series, the state space model, the neural networks and the grey forecasting. From our literature review, only some of the traditional time-series methods (e.g., autoregressive models, ARMA, and NNM) [8, 9] and the chaotic forecasting models are used for time-series analysis in the turbulent flow study [10–12]. This study proposes an innovative approach to solve non-linear time-series of turbulence. We also compare some existing time-series analysis methods in general (see Table III) [26, 27].

First of all, the traditional time-series methods (i.e., moving average, exponential smoothing, decomposition and growth curve methods) are based on the theory of statistics and the parameters of the models are time-invariant. Most of the input and output relationships of these models are nonlinear, except for the moving average method and the additive type of decomposition model. The classic decomposition models and the Winter's exponential method are suitable for

Table III. Comparison of 14 time-series methods.

Model	Number of observations	Theory	State	Equation period	Time problems	Appropriate
ARIMA	–	probability	nondynamic	linear	short	trend
Simple exponential smoothing	5–10	probability	nondynamic	nonlinear	short	stationary
Holt's exponential	10–15	probability	nondynamic	nonlinear	short & medium	trend
Winter's exponential	at least 5	probability	nondynamic	nonlinear	short & medium	trend & seasonality
Decomposition	2 cycle time	probability	nondynamic	linear and nonlinear	short & medium	trend & seasonality
Growth curve	7–10	probability	nondynamic	nonlinear	long	s-curve, technology forecasting
Box–Jenkins	at least 50	probability	nondynamic	linear	short	trend & seasonality
NNM	–	probability	nondynamic and dynamic	nonlinear	short	trend & seasonality
Kalman filtering	little	conditional probability	dynamic	linear and nonlinear	short	trend & seasonality
Deterministic chaos	–	differential or difference equation	dynamic	nonlinear	short	nonstationary
Fuzzy time-series (Song et al. and Chen)	–	fuzzy relation	nondynamic	–	short	trend
Fuzzy regression	–	possibility	nondynamic	linear and nonlinear	medium & long	trend
Fuzzy piecewise regression with automatic change-point	–	fuzzy relation	dynamic	linear and nonlinear	medium & long	trend
Neural network (back-propagation)	–	gradient descent	dynamic	nonlinear	medium & long	trend & seasonality
Grey forecasting	at least 4	grey theory	nondynamic and dynamic	nonlinear	short	trend

forecasting time-series with growth trend and seasonality [28, 29]. However, the moving average method and the Holt's smoothing method are not suitable for seasonal time-series. The theoretical foundation of the Box–Jenkins model is based on the probabilistic distribution of statistics and the fact that the relationship between input and output is precise linear functional form [30]. It is widely used for short term forecasting, especially for time-series with trend and seasonality [28]. The disadvantage of this model is its need of at least 50 or more historical data required. A Nearest Neighbor Method (NNM) was developed to predict riverflow by Kember et al. [9]. They applied the nonlinear dynamics into the parsimonious three parameters NNM prediction. However, the stochastic approach is not satisfied in turbulent flow prediction with chaotic dynamics.

For the dynamic models, Kalman filtering methods and chaotic models are the most popular ones. The Kalman filtering method used the conditional probability concept. The parameters will change when a new data becomes known. This means the parameters are time-variant. Recent studies applied fuzzy set theory or neural network to Kalman filtering [31] in prediction. On the other hand, systems which are nonlinear, nonequilibrium, deterministic, and dynamic, incorporate randomness, are sensitive to initial conditions, and have strange attractors are said to be chaotic. Deterministic chaos can provide guidance for short-term predictions under certain circumstances when the attractors have only a few dimensions while it is unable to make long term predictions [32]. The chaotic models can divulge a good insight of the dynamics and are used widely in most turbulent time-series research [10–12].

Another approach for consideration is the fuzzy time-series model. The Chen, Song and Tseng fuzzy time-series use the theory of fuzzy relation [33–38] and their models can be used to deal with forecasting problems in which historical data are linguistic values. The thought behind these models is based on the fuzzy relational equations and approximate reasoning. This fuzzy time-series idea in additional to nonlinear chaotic concept consists of the fundamental outline of this paper. With respect to fuzzy regression method, it emphasizes on the ambiguity and the indefinite nature of a system, while the traditional regression analysis emphasizes on the randomness. This method allows the relationships between the response and the predictor variables to be ambiguous. This uncertainty relationship can be represented by fuzzy parameters defined as fuzzy sets, or more specifically, the fuzzy numbers [17]. Furthermore, the inferred values of the response are also fuzzy because they are derived from fuzzy parameters. The details of this method are provided in Sections 2 and 3 of this paper. The fuzzy piecewise regression analysis with automatic change-point rather than traditional regression method consists in its piecewise dynamical prediction. It applies piecewise change-point analysis to predict the time-series dynamically. The prediction results shall rely on the dynamic of the system. In the other prediction methods, neural networks rely on training data exclusively to generate a forecast. The theoretical advantage of neural networks as a forecasting tool is that the relationship does not need to be specified in advance since the method can establish relationships through a learning process. Also, neural networks do not require any assumption about underlying population distribution. They are especially valuable in cases where inputs are highly correlated, missing, or nonlinear systems [36]. The back-propagation model uses gradient descent to come up with the best solution. The grey forecasting method is based on the grey theory. It can be used in which only little data is collected, as few as four observations are enough [37], to estimate the outcome of an unknown system. And it can use a first order differential equation to characterize a system.

Therefore, only a few discrete data is sufficient to characterize such an unknown system. The neural networks and grey forecasting are two strategies that can be combined with other models to improve prediction in turbulent time-series model in the future study.

Under these circumstances, the use of the fuzzy piecewise regression model to predict the nonlinear time-series of turbulent flows with automatic change-point detection is more satisfactory than the models in other published papers. There are several advantages that the fuzzy piecewise regression model with automatic change-point detection has which make it appear to be the most appropriate tool:

(i) The method can deal with the problems simply using the piecewise nonlinear time-series regression to predict the time-series trends with complex dynamics intrinsically. In fact, when the dynamics of turbulence reveals ambiguous and uncertain observed data, the proposed fuzzy piecewise regression analysis method can capture the instant variation by the change-points.

(ii) The method can detect the change-points of nonlinear time-series regression automatically and is more robust than conventional fuzzy regression. The conventional fuzzy regression is sensitive to outliers. Based on a piecewise concept, the proposed method can deal with outliers by segmenting the data automatically.

(iii) The required number of observations for the forecasting is less than the large amount of observations in the other publication. It can be shown more efficient in dealing with nonlinear time-series data by using the fuzzy piecewise analysis than other methods (e.g., chaos dynamics, statistical method, autoregressive moving average, etc.). The proposed method is useful in piecewise time-series forecasting.

A new scope of this research in time-series for turbulent analysis is to develop an adjoint method composed of the dynamic chaotic theorem and fuzzy piecewise regression in order to make long-term prediction, which can combine both the advantages of chaos theorem in short-time precise prediction and the fuzzy piecewise regression in the long-term trend. An actual potential of using this model is the time-series prediction of high-dimensional dynamical system.

5. Conclusions

This research uses general fuzzy piecewise regression analysis with automatic change-point detection to predict the nonlinear time-series of near-wall turbulent flows. Fuzzy piecewise possibility and necessity regression models are used when the function behaves differently in different parts of the range of crisp input data. The nonlinear prediction method performs better in the forecast than the statistical method. In this method, the observed data can be points or intervals and the observed data are nonlinear. The positions of change-points and the fuzzy regression possibility and necessity models can be simultaneously obtained. Based on piecewise characteristics and concepts, we can deal with the prediction of nonlinear situations in the near-wall turbulence time-series by using fuzzy piecewise regression successfully. This paper also overcomes the problem in searching change-point technique by using automatic detection.

In order to show and explain the practicality and usefulness of this model, an example of predicting the nonlinear behavior of near-wall turbulence time-series is taken as a validated model by subjectively taking eleven sample points of the time-series experimental data from Figure 1 [12]. We use two forms to test this model: (a) linear piecewise time-series and (b) quadratic piecewise time-series. According to these results of practical applications, the proposed method is appropriate and appears to be useful in searching the change-points of nonlinear situations of near-wall turbulence for predicting the turbulence time-series automatically as well as in a fuzzy environment.

Appendix

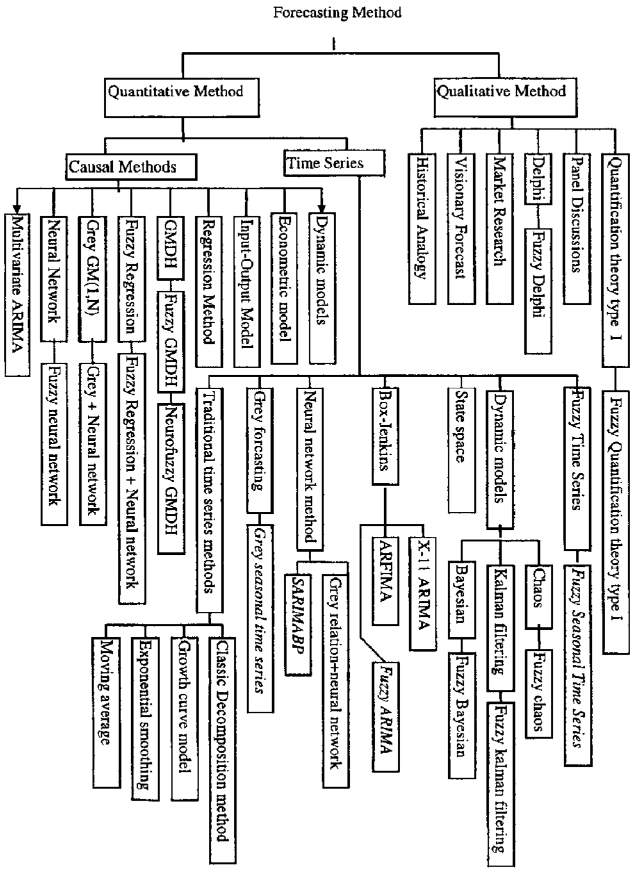

Figure A.1. The categories of forecasting models.

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