Direct Numerical Simulation
of
Wind-Wave Generation Processes

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Direct Numerical Simulation of Wind-Wave Generation Processes

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Outline

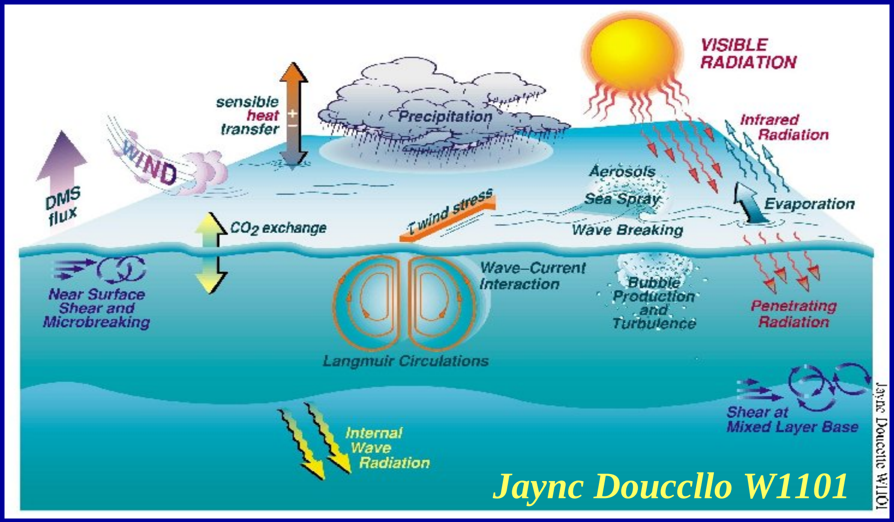
1. *Why study wind-wave generation processes?*
2. How to develop an air-water coupled model?
3. What we observe?
4. Wave growth types
5. Compare with previous studies

Overview

The system of atmosphere and ocean is not independent

<u>Wind Waves</u> :

Wind-generated waves are the most visible signature of air-sea interaction and play a major influence on the momentum and energy transfer across the interface.



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Overview

The mechanisms that generate these surface waves are still open issue due to

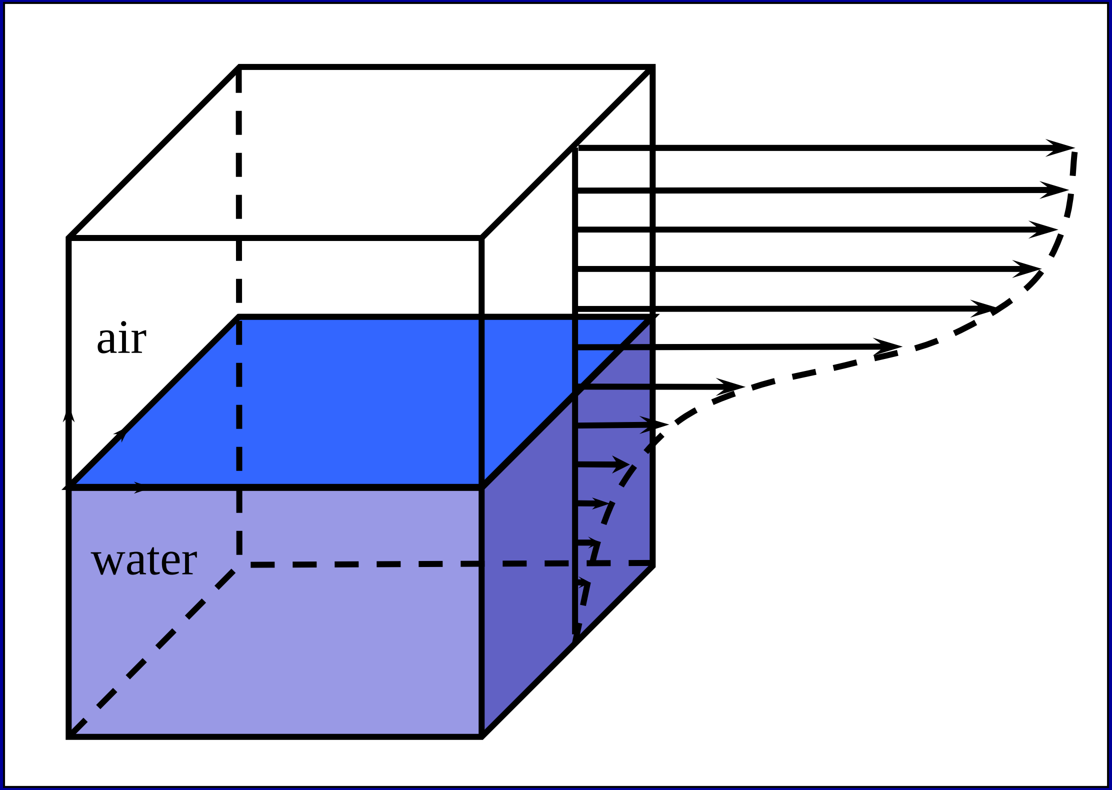
(1) Difficulties in obtaining a dataset from laboratory and field measurements that records the time evolution of motions in both atmosphere and ocean domains

(2) Mathematical difficulties in dealing with highly turbulent flows over complex moving surfaces

(3) Lack of a suitable coupled model to simulate turbulent flows in both atmosphere and ocean simultaneously

The Purpose of this Research

- **Develop an air-water coupled model**
- **Study the wind-wave generation processes (laboratory waves)**



Outline

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2. *How to develop an air-water coupled model?*
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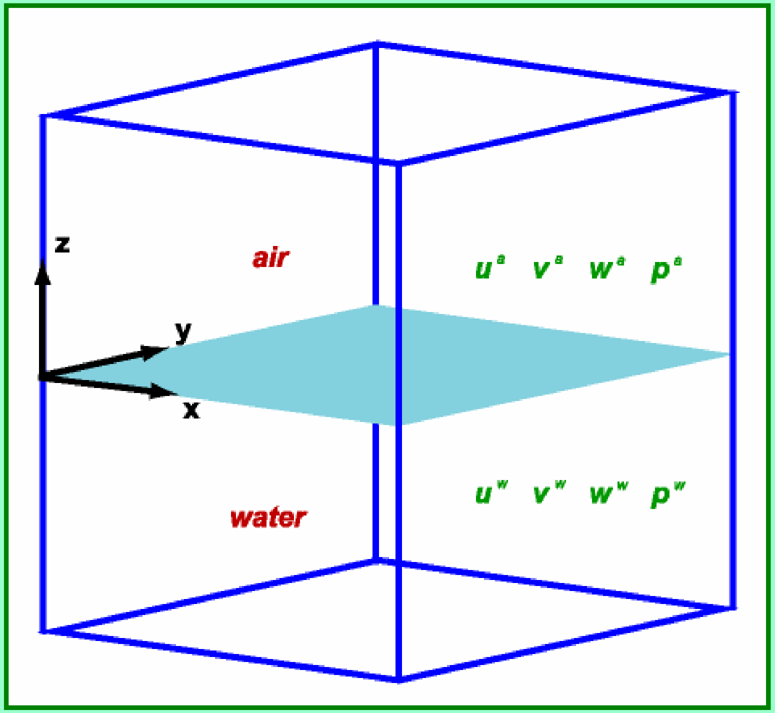
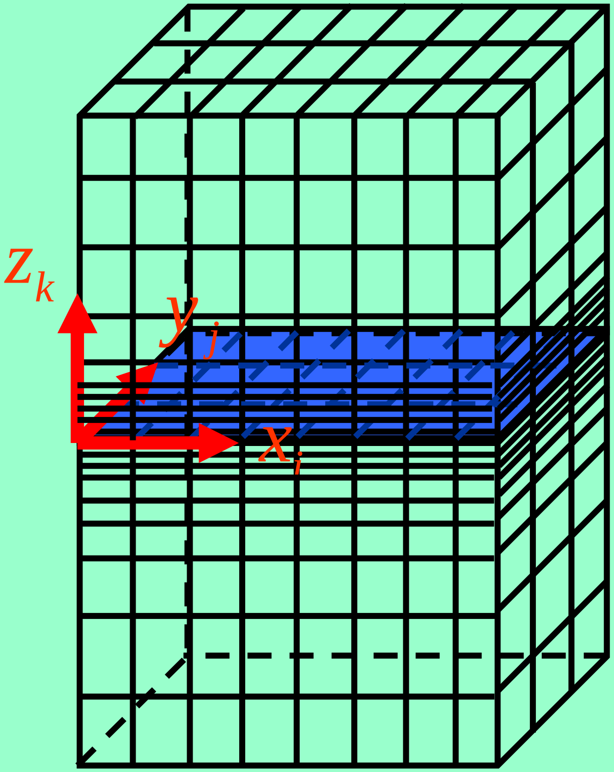
Direct Numerical Simulation

DNS numerically solves the Navier-Stokes equation subject to boundary conditions and hence such simulated flow fields contain no uncertainties other than numerical errors.



Domain size : $24 \times 24 \times 8\ \text{cm}^3$



Grid points : $2 \times (64,64,65)$

Differencing Schemes

Spatial Differencing :

horizontal: pseudo-spectral method

vertical: second order finite differencing

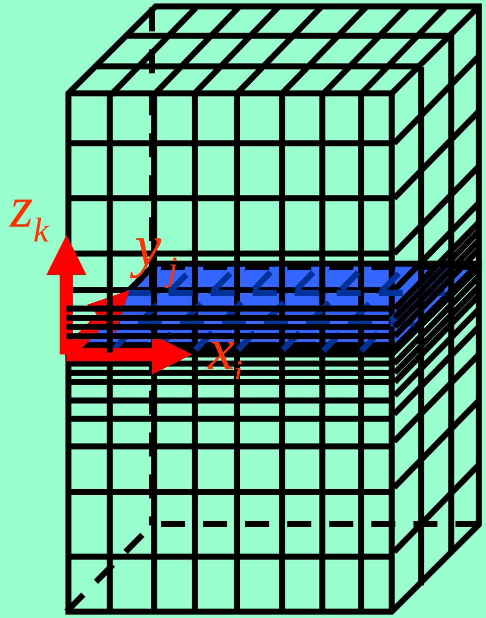
Time Differencing :

second order Runge-Kutta scheme

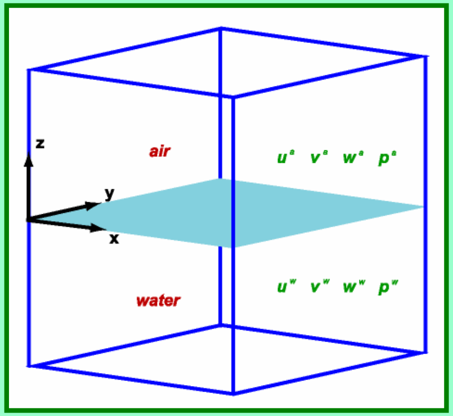
Grid System :

stretching grid system

high resolution near interface



Boundaries & Boundary Conditions



For

4 side walls :

periodic boundary conditions

lower boundary :

free-slip boundary conditions

upper boundary :

a constant velocity is imposed

interfacial boundary : (at air-water interface)

The conditions for interfacial boundary are

1. ***Velocity is continuous***
2. ***Stress is continuous***

Problem Formulation of Two-Phase Coupled Flow

Governing Equations :

$\ell = a$ air

$\ell = w$ water

$$\nabla \cdot \vec{u}_\ell = 0$$

$$\frac{\partial \vec{u}_\ell}{\partial t} + \left(\vec{u}_\ell \cdot \nabla\right)\vec{u}_\ell = -\nabla p_\ell + \frac{1}{\mathrm{Re}_\ell}\nabla^2 \vec{u}_\ell$$

$$\vec{u}_\ell = \left(u_\ell, v_\ell, w_\ell\right)$$

Interfacial Boundary Conditions : (linearized)

continuity of velocity

$$u_a = u_w, \qquad v_a = v_w, \qquad w_a = w_w$$

continuity of normal stress

$$P_w - \frac{\rho_a}{\rho_w} P_a = f_a\left(u_a, v_a, w_a, \eta\right) - f_w\left(u_w, v_w, w_w, \eta\right)$$

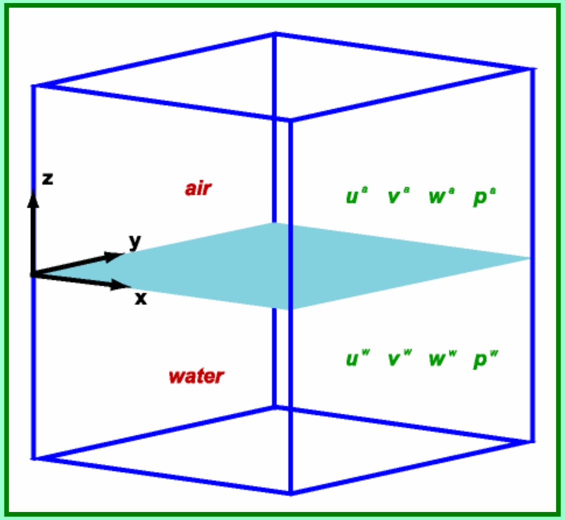
continuity of shear stress

$$\left(\frac{\partial u_a}{\partial z} + \frac{\partial w_a}{\partial x}\right) = \frac{\mu_w}{\mu_a}\left(\frac{\partial u_w}{\partial z} + \frac{\partial w_w}{\partial x}\right)$$

$$\left(\frac{\partial v_a}{\partial z} + \frac{\partial w_a}{\partial y}\right) = \frac{\mu_w}{\mu_a}\left(\frac{\partial v_w}{\partial z} + \frac{\partial w_w}{\partial y}\right)$$

Kinematic free surface B. C.

$$\frac{\partial \eta}{\partial t} = w - \frac{\partial u \eta}{\partial x} - \frac{\partial v \eta}{\partial y}$$

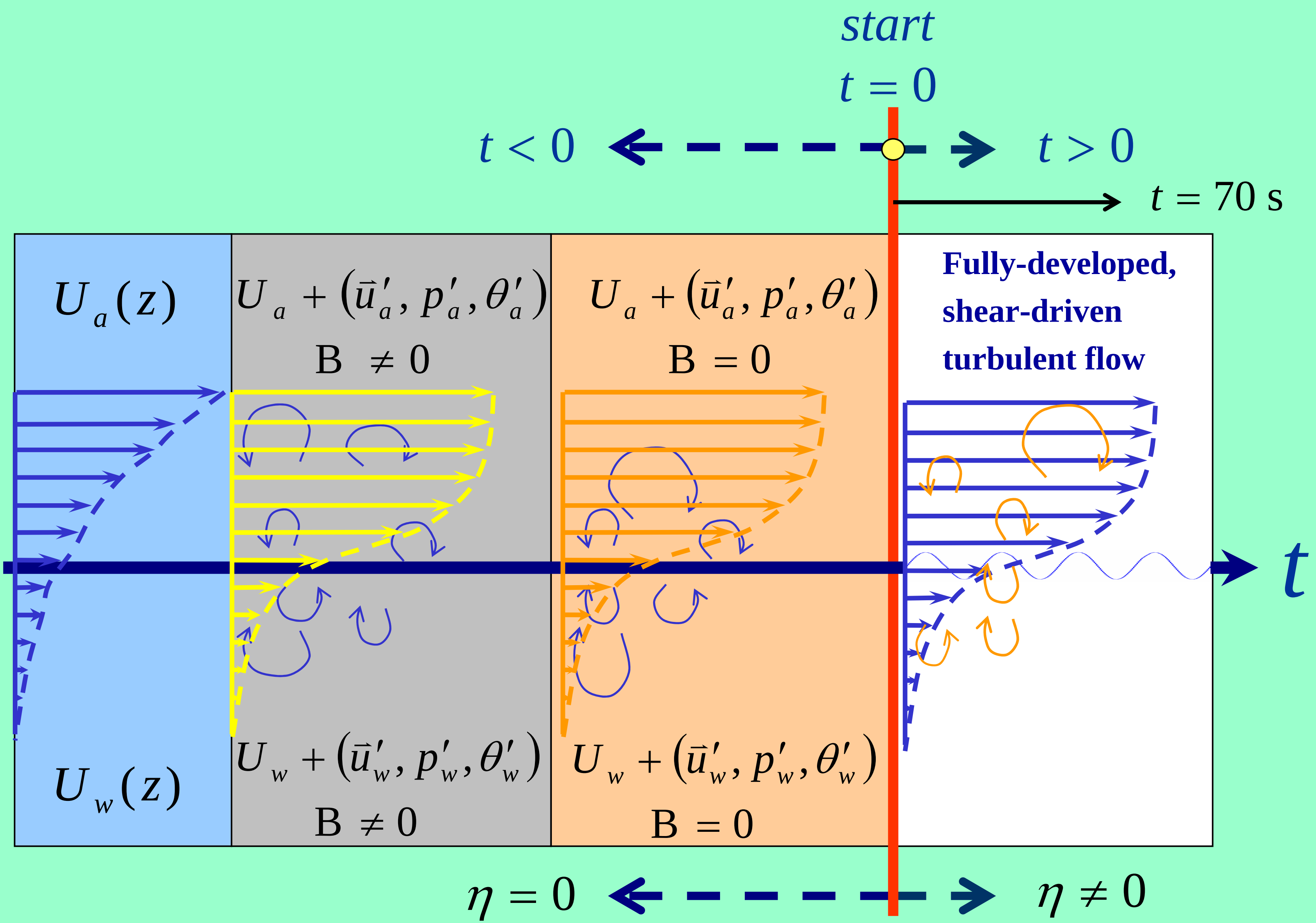


- Interfacial boundary conditions are linearized

- Limitation of the air-water coupled model

  → only for small amplitude waves

Initialization



start

$t = 0$

$t < 0$

$t > 0$

$t = 70\ \mathrm{s}$

$U_a(z)$

$U_a + (\vec{u}'_a, p'_a, \theta'_a)$

$B \neq 0$

$U_a + (\vec{u}'_a, p'_a, \theta'_a)$

$B = 0$

Fully-developed, shear-driven turbulent flow

t

$U_w(z)$

$U_w + (\vec{u}'_w, p'_w, \theta'_w)$

$B \neq 0$

$U_w + (\vec{u}'_w, p'_w, \theta'_w)$

$B = 0$

$\eta = 0$

$\eta \neq 0$

Outline

1. **Why study wind-wave generation processes?**
2. **How to develop an air-water coupled model?**
3. ***What we observe?***
4. **Wave growth types**
5. **Compare with previous studies**

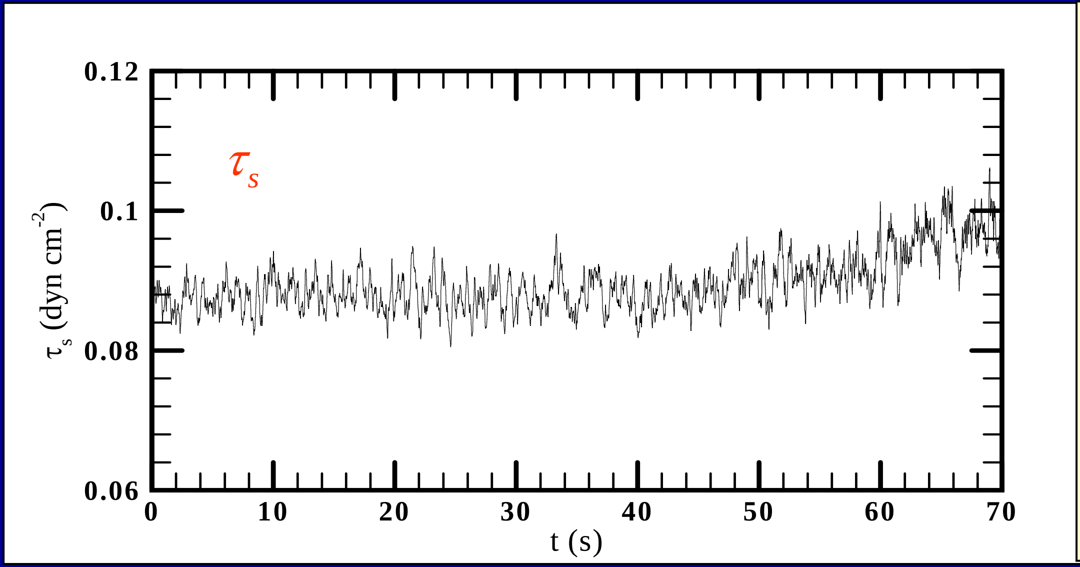
Mean wind stress at the interface

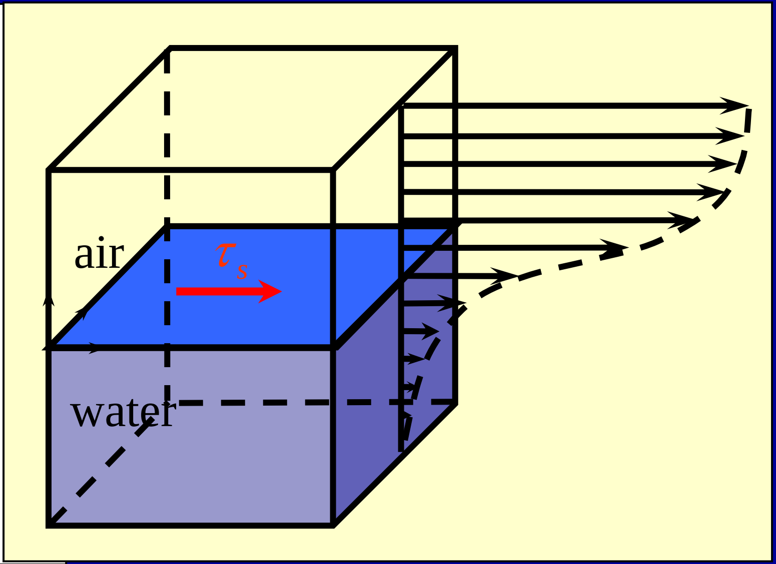
- $t < 50$ s : $\tau_s \sim$ constant

  ⟶ reached a statistically quasi-steady state

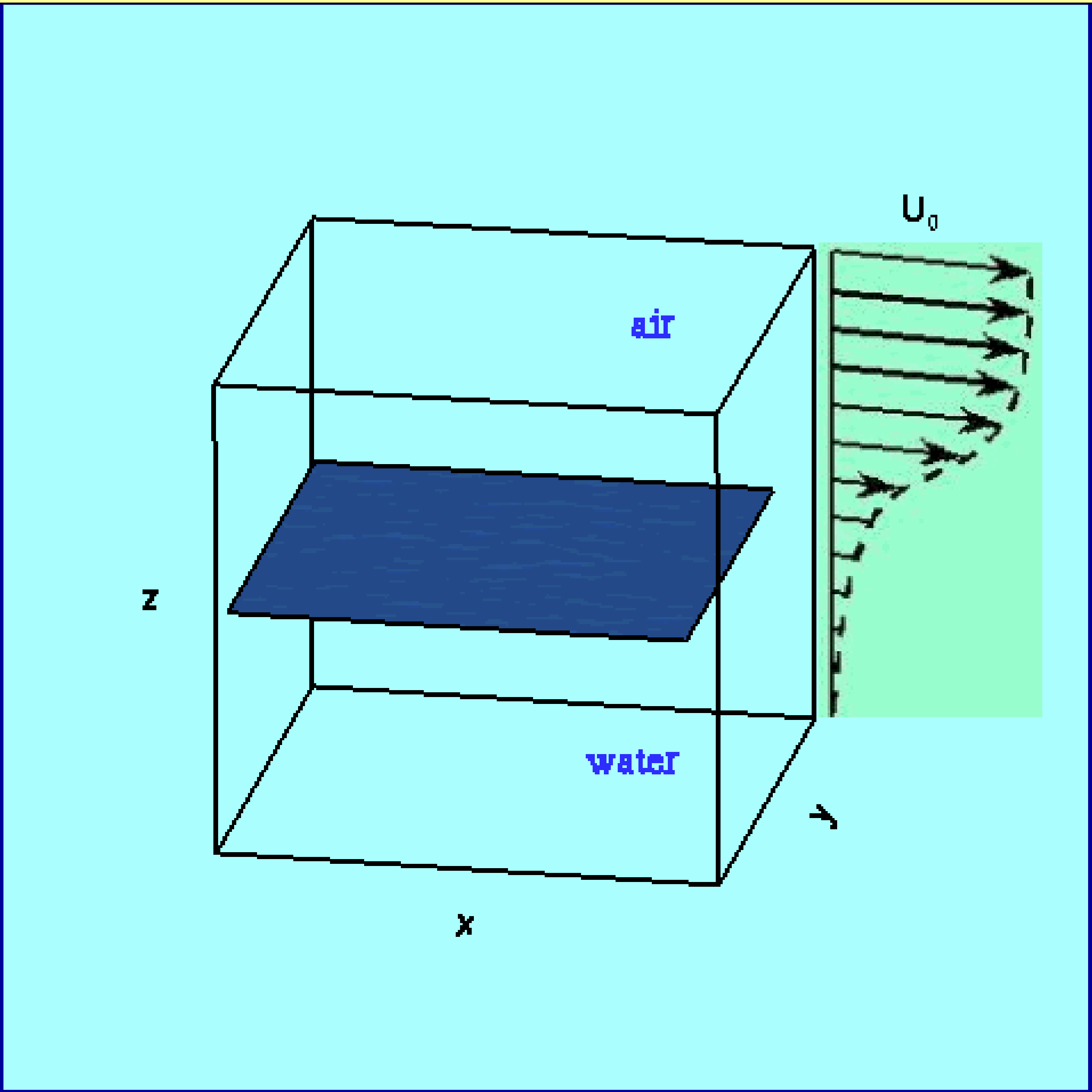
- $t > 50$ s : τ_s increases with time

  ⟶ increases due to the growth of surface waves





Wind-Wave Generation Processes (*t*=0~70 s)



When wave amplitude changes, what will be the behavior of the flow fields above and below the interface?

$$\eta \leftrightarrow \; u, \;\; v, \;\; w, \;\; p$$

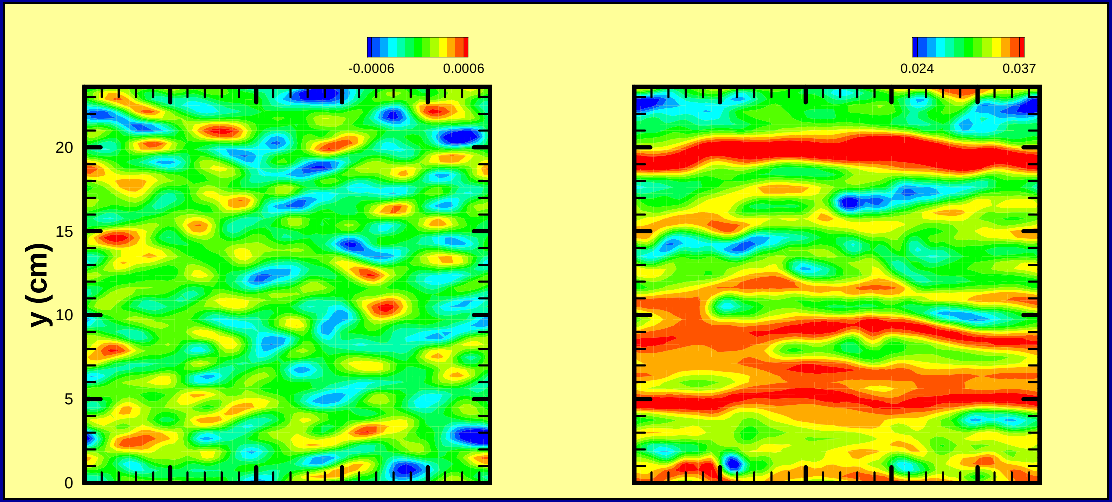
(surface wave elevation)

Waves & Streamwise Velocity at the Interface

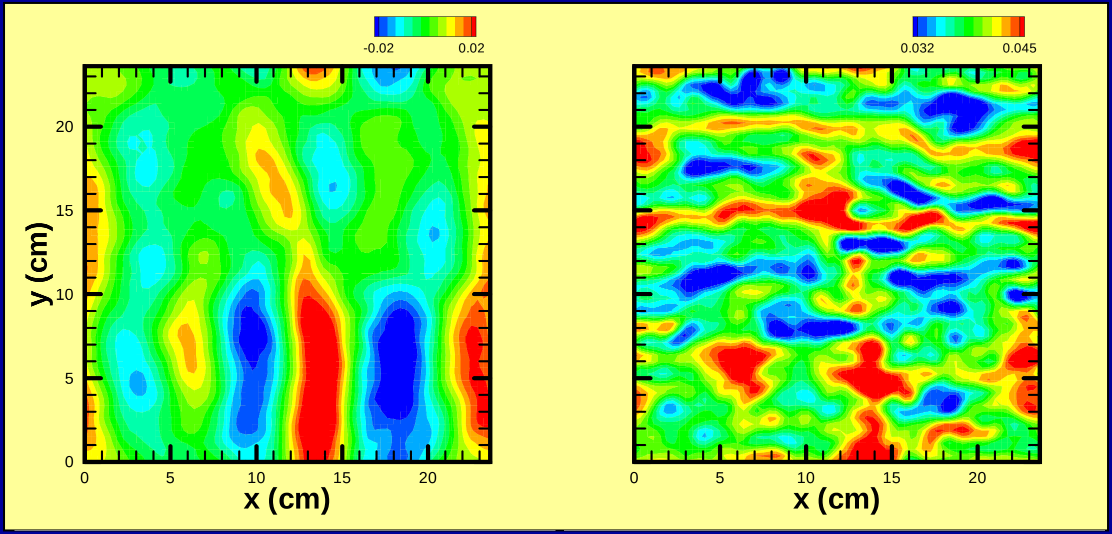
$\eta(x, y)$

$u_w(x, y, z = 0)$

$t = 2.6\,\mathrm{s}$



$t = 64\,\mathrm{s}$



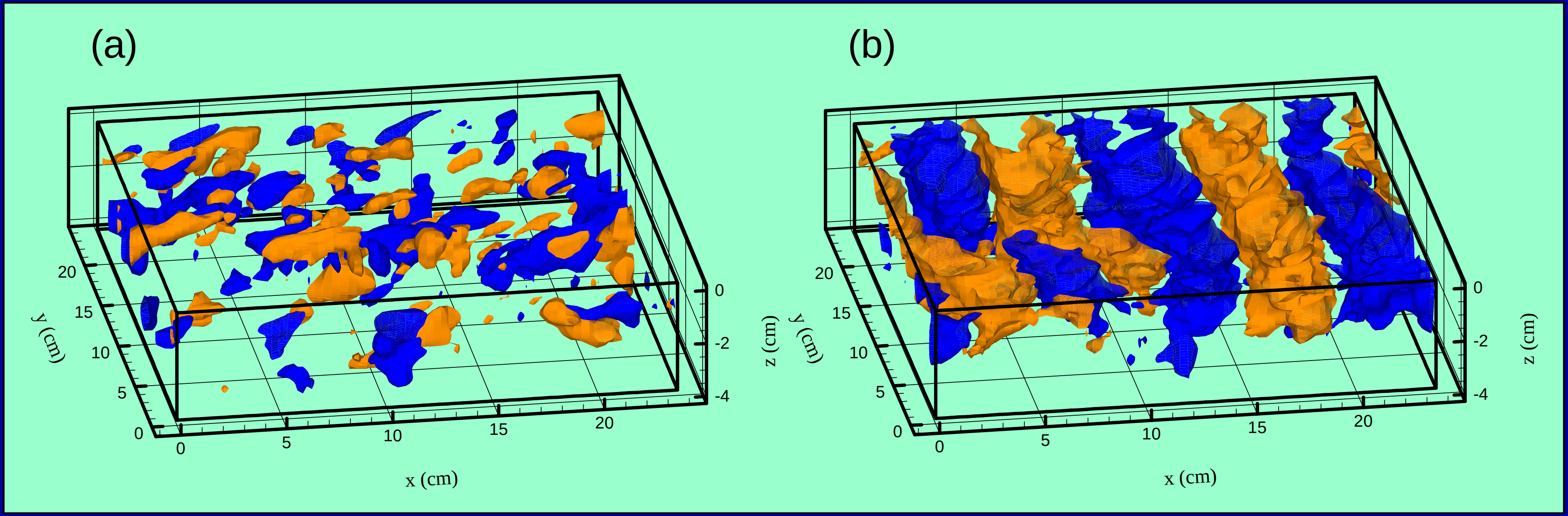
w' in the Water

- **At shear-dominated stage (t=16 s) :**

  the distribution of updrafts and downdrafts is irregular

- **At wave-dominated stage (t=68 s) :**

  the vertical velocity field aligns with waves



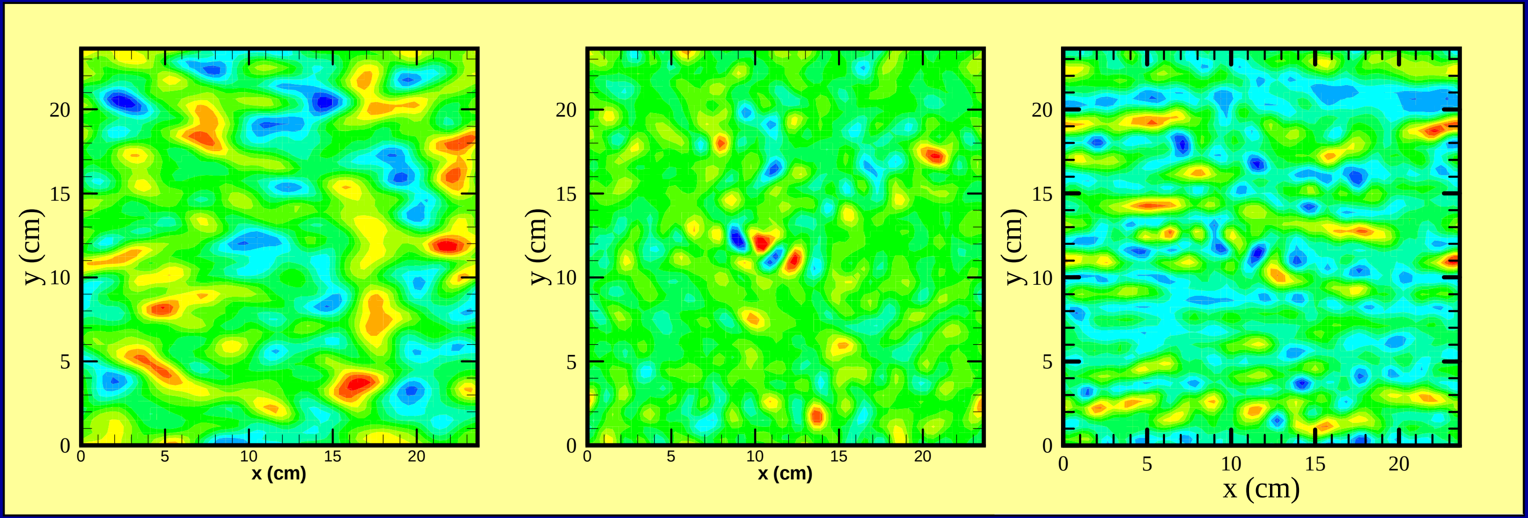
$t = 16\,\mathrm{s}$

$t = 68\,\mathrm{s}$

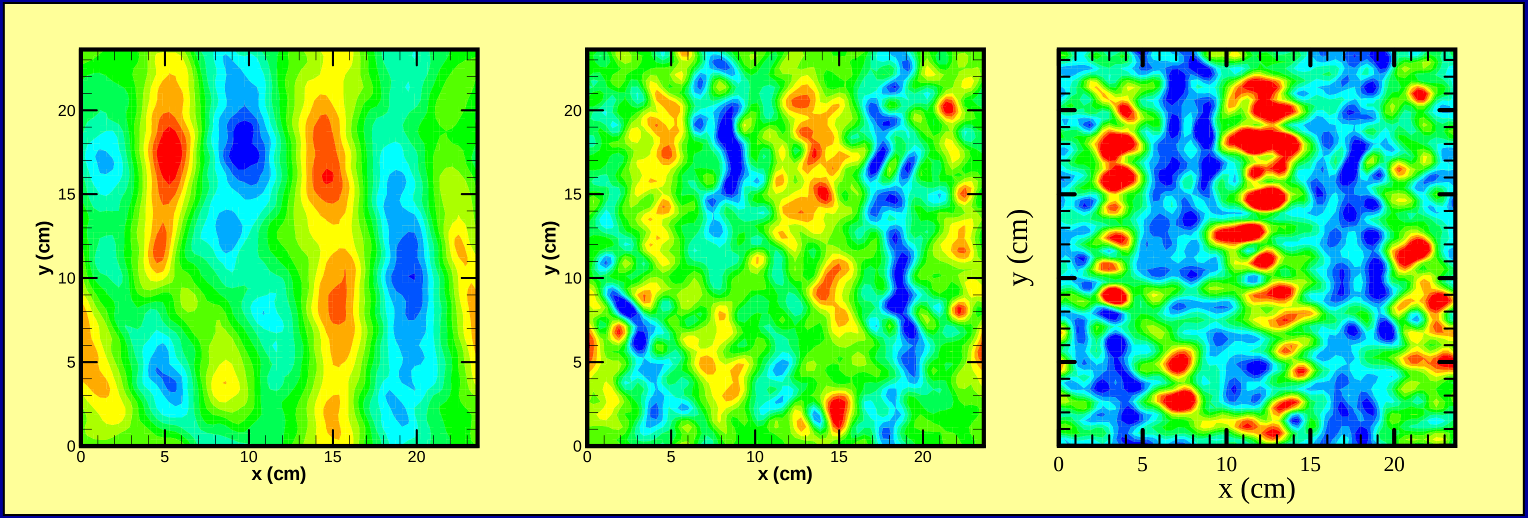
Waves,
Surface Pressure of the Air & Shear Stress Fluctuations

$\eta(x, y)$ $p'_a(x, y, z=0)$ $\tau'_s(x, y, z=0)$

$t = 16\,\mathrm{s}$



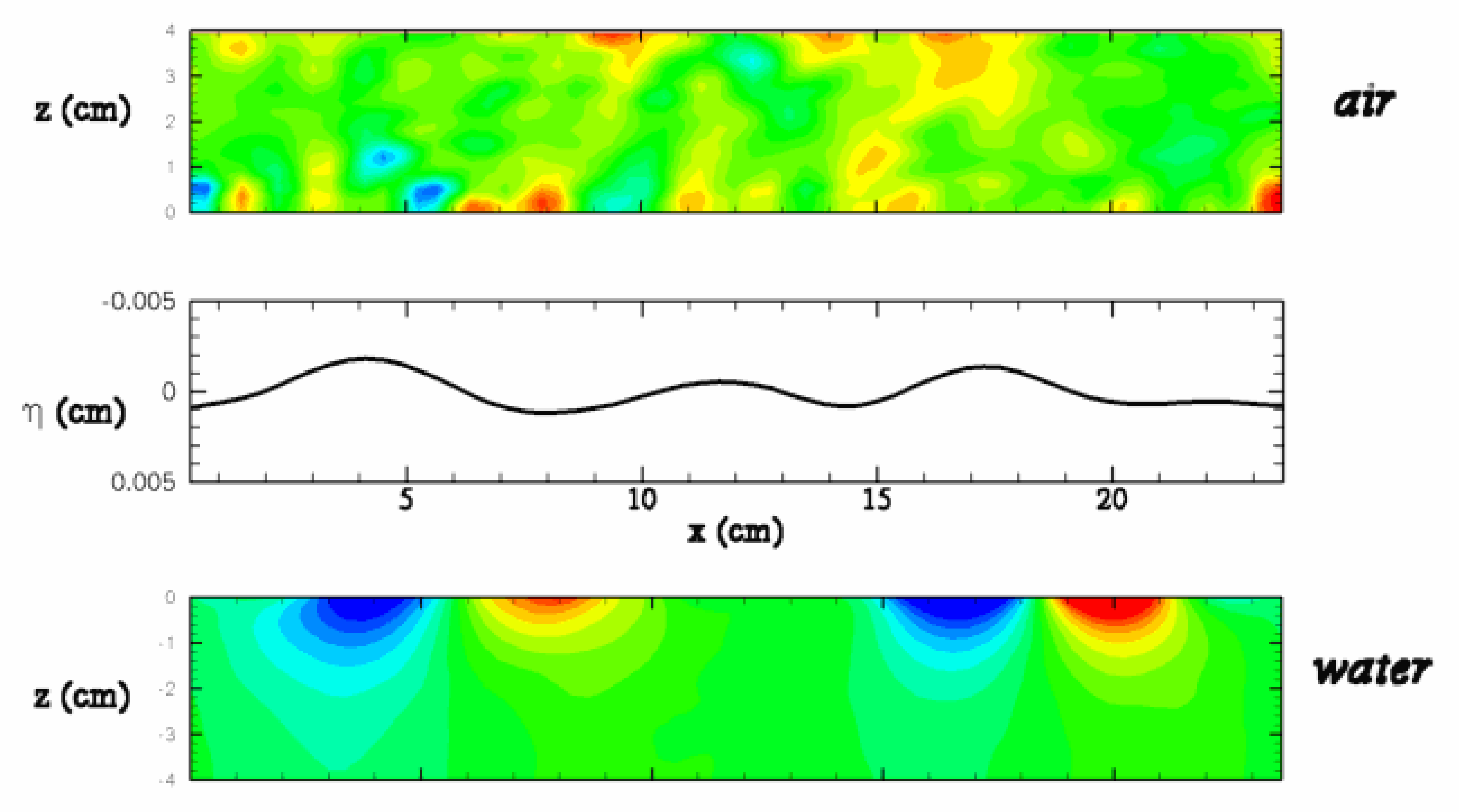
$t = 68\,\mathrm{s}$



Waves & Pressure Fluctuations (a vertical cross-section)

At early stage

$t = 16 \sim 16.5\,\mathrm{s}$

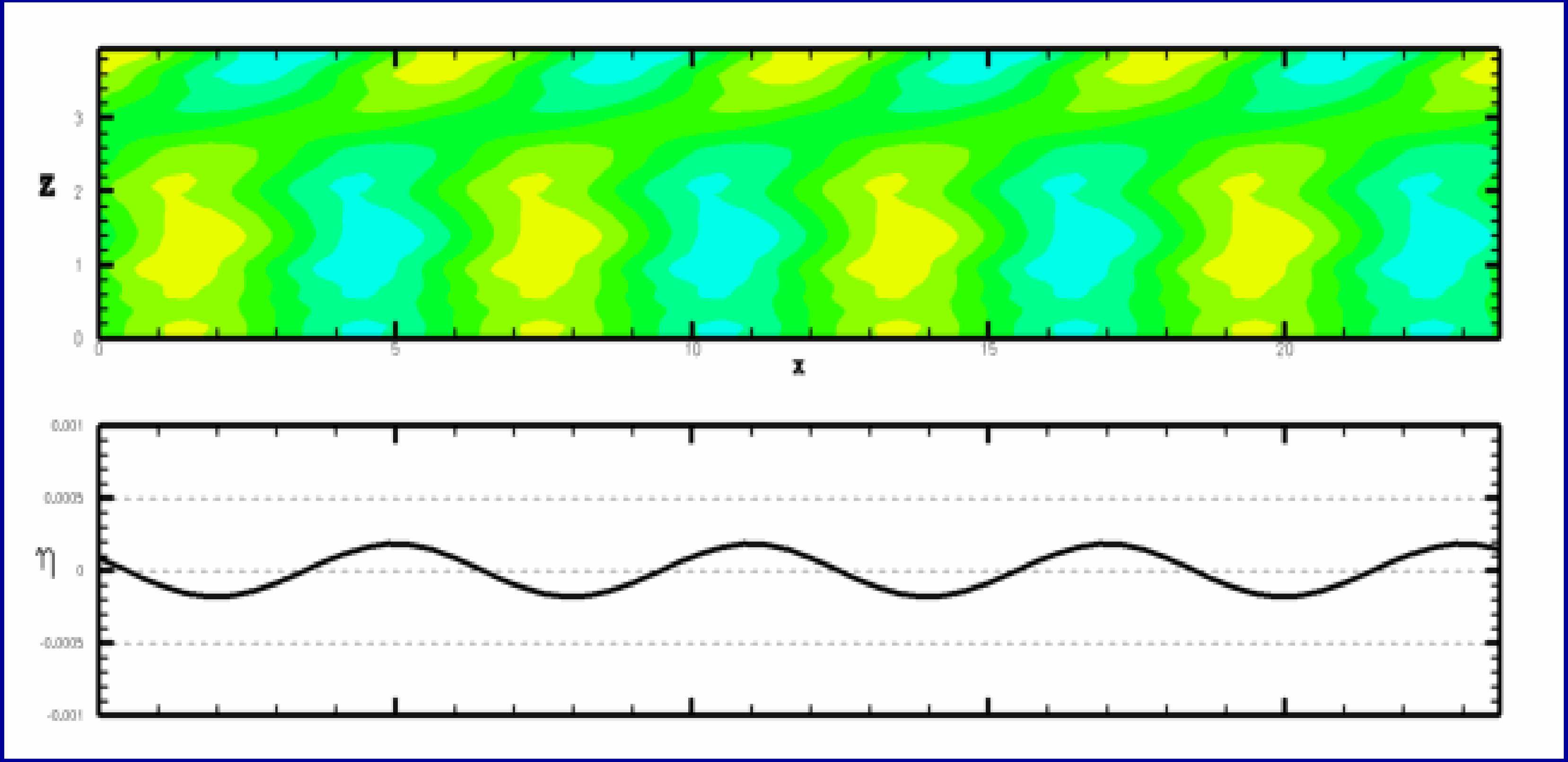


Waves & Pressure Fluctuations (a vertical cross-section)

For one wave component : $\left(k_x, k_y\right) = (1., 0.)\,\mathrm{cm}^{-1}$

At early stage : $t = 16 \sim 16.5\,\mathrm{s}$

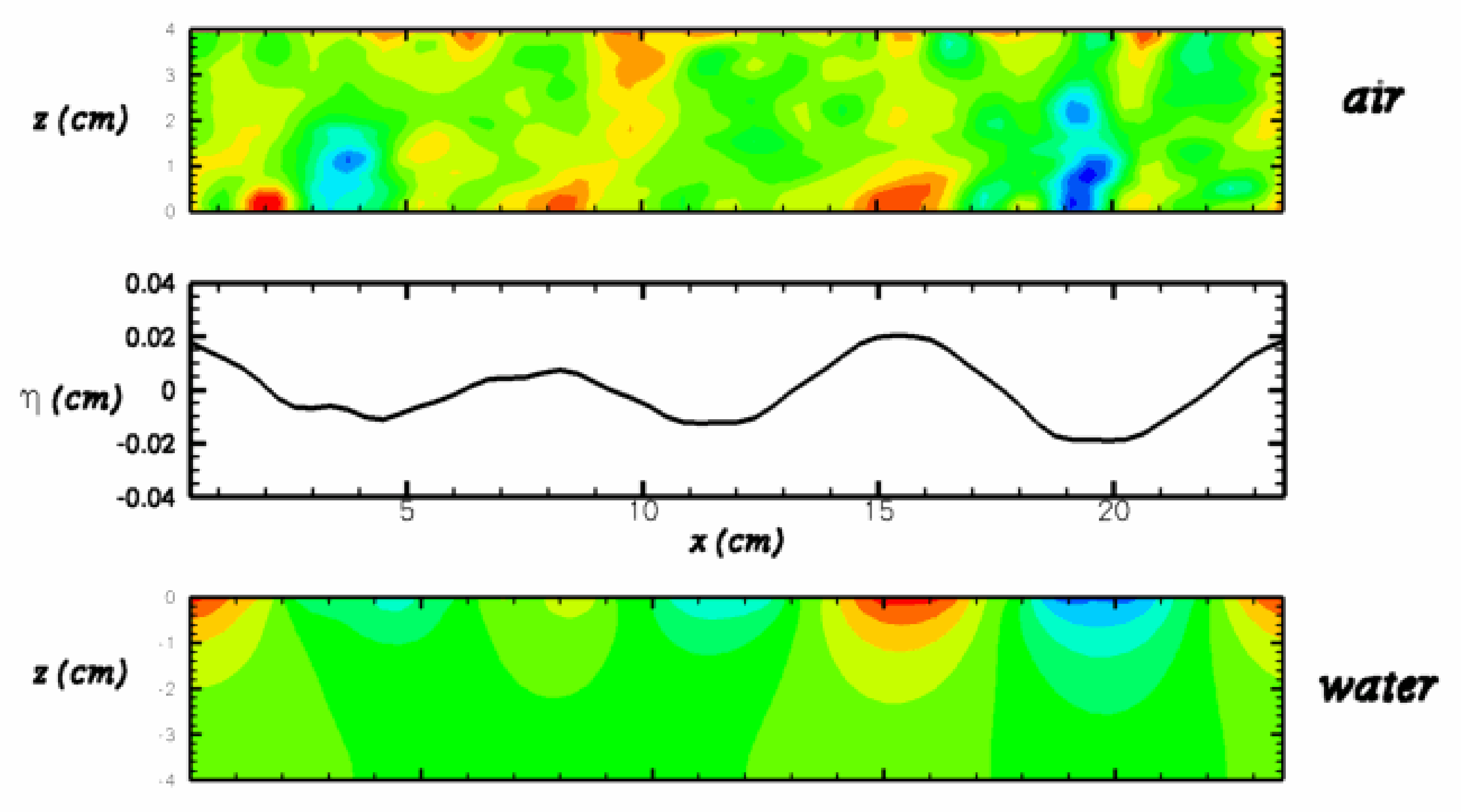
air



Waves & Pressure Fluctuations (a vertical cross-section)

At late stage

$t = 50.5 \sim 51\,\mathrm{s}$



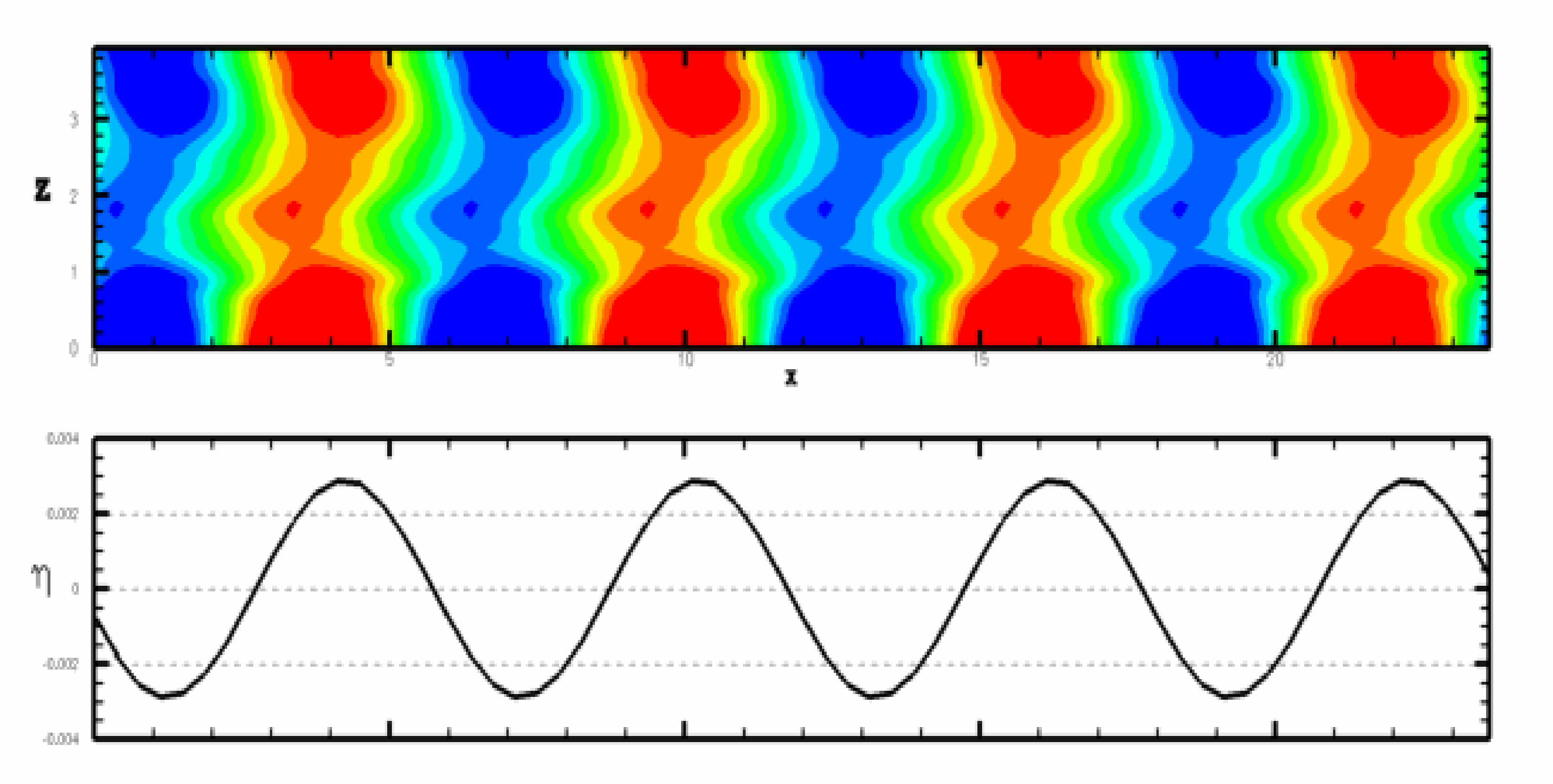
Both domains are strongly influenced by waves

Waves & Pressure Fluctuations (a vertical cross-section)

For one wave component : $\left(k_x, k_y\right) = \left(1., 0.\right)\,\mathrm{cm}^{-1}$

At late stage : $t = 50.5 \sim 51\,\mathrm{s}$

air



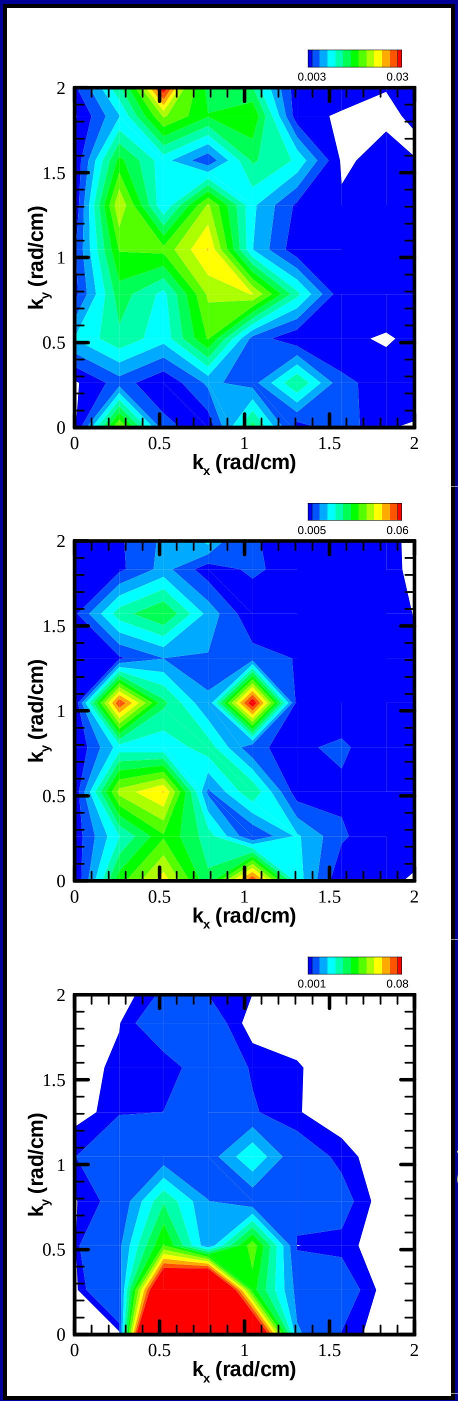
Spectra of Wave Energy

Wavelength ~ 8-12 cm

Wave frequency ~ 37 s^{-1}

→ Satisfy the dispersion relationship

	Wave number (cm^{-1}) $\kappa = (k_x, k_y)$	$\Phi(\eta)/\overline{\eta}^2$
$t \sim 16\,\mathrm{s}$	**(0.26, 1.05)** **(1.05, 1.05)** **(1.05, 0.)** **(0.52, 0.52)** **(0.52, 0.)**	**7.1 %** **5.6 %** **5.3 %** **4.5 %** **4.1 %**
t ~ 66 s	**(0.78, 0.)** **(0.78, 0.26)** **(0.52, 0.)** **(0.52,0.26)** **(1., 0.26)**	**28 %** **24.7 %** **24.5 %** **7.2%** **3.3%**



$t = 3\,\mathrm{s}$

$t = 16\,\mathrm{s}$

$t = 64\,\mathrm{s}$

Dominated waves are different

Outline

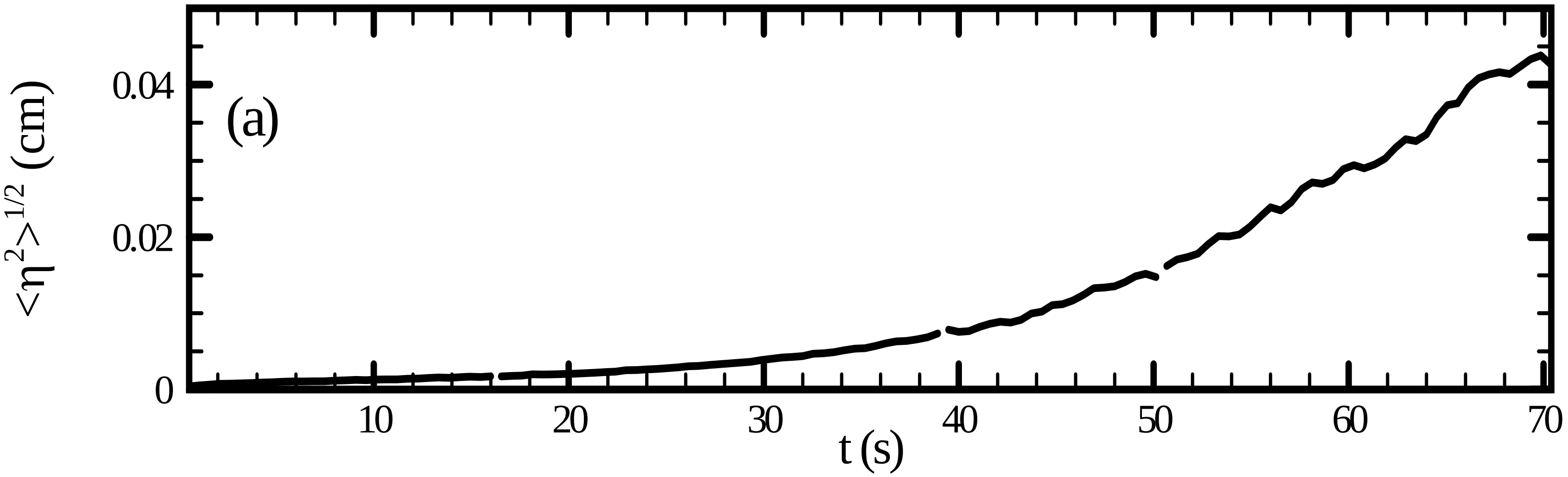
1. Why study wind-wave generation processes?
2. How to develop an air-water coupled model?
3. What we observe?
4. *Wave growth types*
5. Compare with previous studies

Some theoretical studies suggest that wave growth process can be separated into

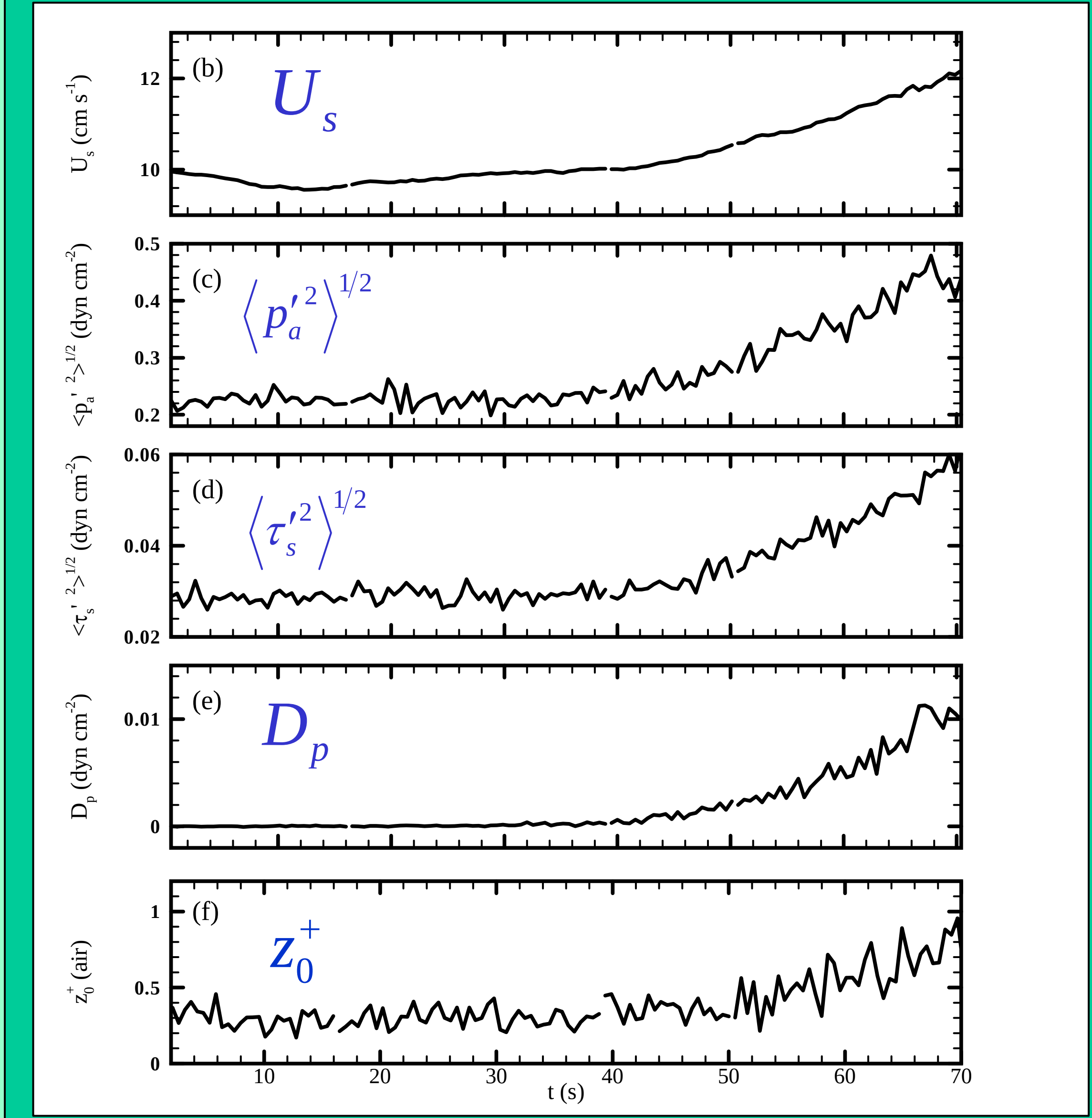
1. **Linear (waves grow slowly)**
2. **Exponential (waves grow quickly)**

Time Evolution of Wave Amplitude

$$\begin{cases} t < 40\,\mathrm{s} & \text{waves grow slowly} \\ t > 40\,\mathrm{s} & \text{waves grow quickly} \end{cases}$$



Time Evolution of Some Parameters at the Interface

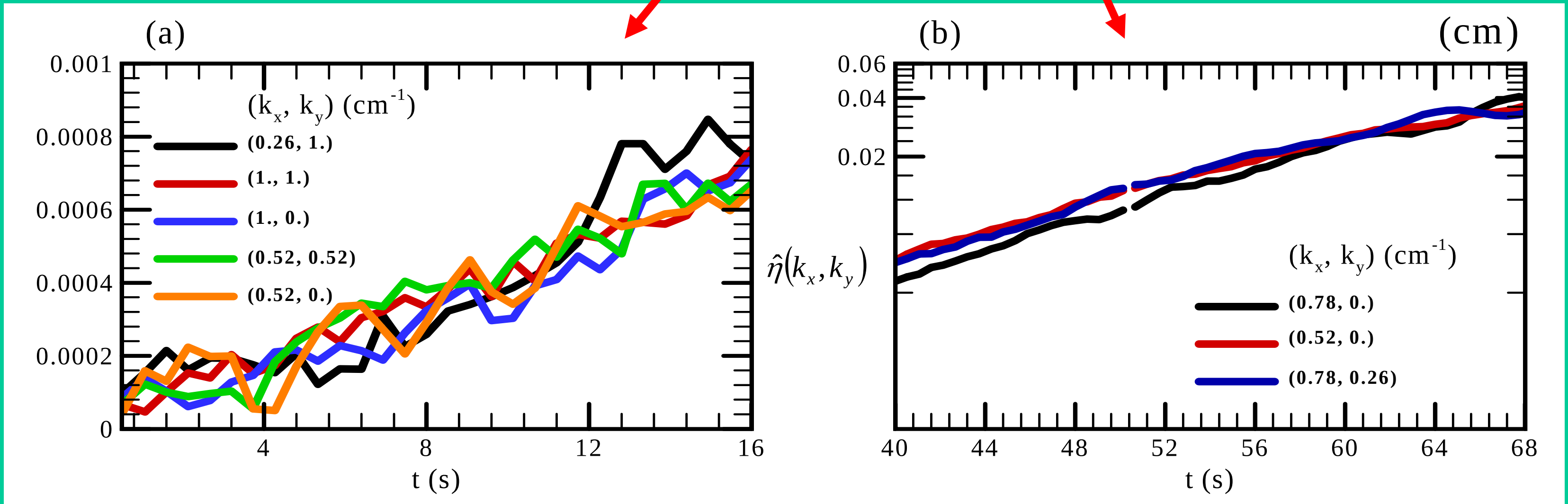


Waves Growth Types

Linear : **t < 16 s**

Exponential : **t > 40 s**

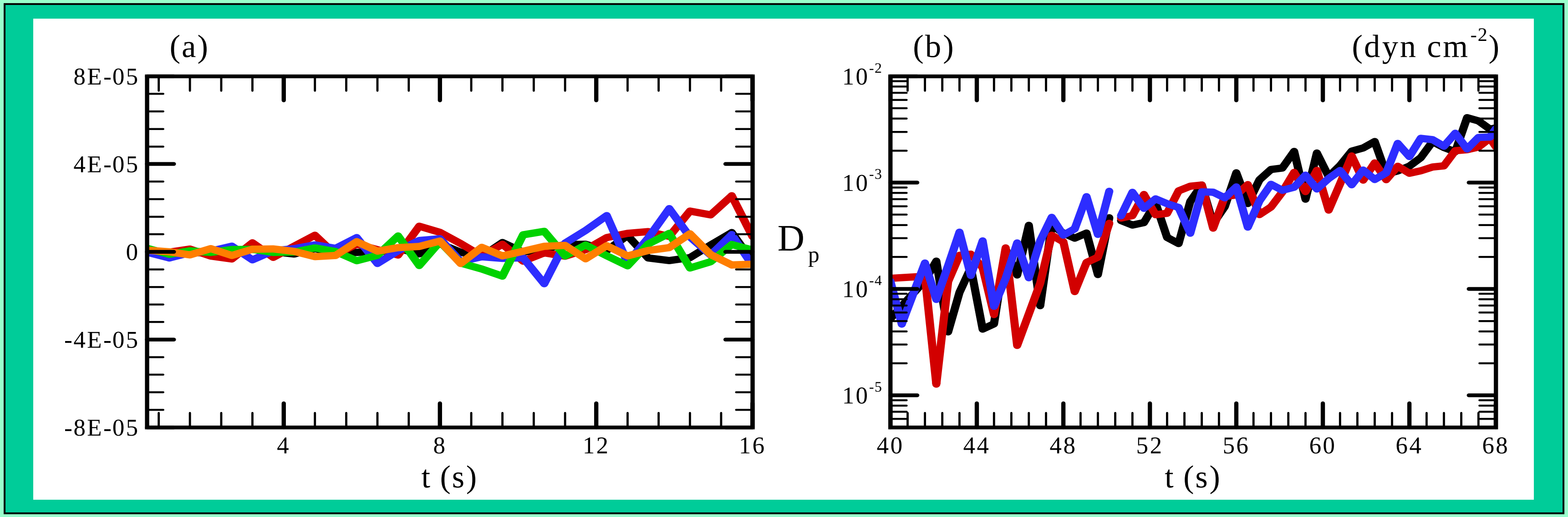
	Wave number **(cm^{-1})** $\kappa = (k_x, k_y)$	$\Phi(\eta)/\overline{\eta}^2$
$t < 16$ s	**(0.26, 1.)** **(1., 1.)** **(1., 0.)** **(0.52, 0.52)** **(0.52, 0.)**	**7.1 %** **5.6 %** **5.3 %** **4.5 %** **4.1 %**
t > 40 s	**(0.78, 0.)** **(0.52, 0.)** **(0.78, 0.26)**	**32 %** **24 %** **21 %**



Form Stress

- Some theoretical studies suggest form stress plays an important role in exponential wave growth stage

$$D_p = \frac{1}{L_x L_y}\int_0^{L_y}\int_0^{L_x} p'_a(\kappa,t)\frac{\partial\eta(\kappa,t)}{\partial x}\,dxdy$$



$t < 16\ \mathrm{s}$ $t > 40\ \mathrm{s}$

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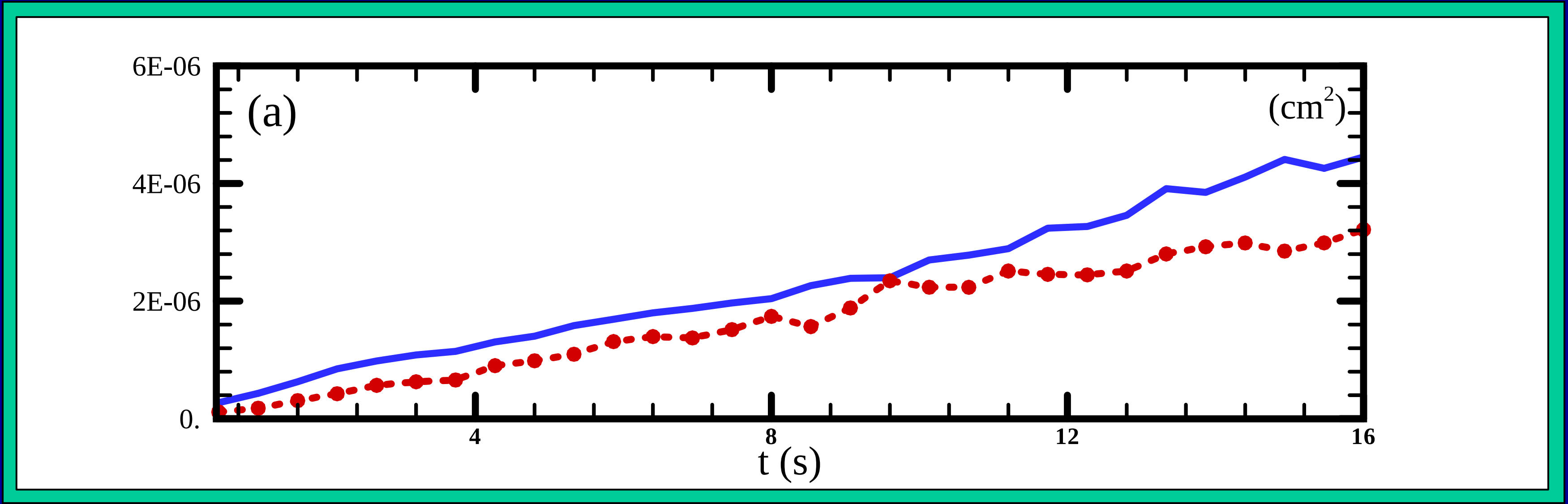
Linear Growth Stage

Phillips (1957) :

the turbulence-induced pressure fluctuations in the air are responsible for the birth and early growth of waves

$$\overline{\xi^2} \sim \frac{\overline{p_a'^{\,2}}}{2\sqrt{2}\rho_w^2 g U_c} t \qquad \textbf{\textit{when}} \qquad U_c \sim 18 u_a^*$$



Exponential Growth Stage

Belcher & Hunt (1993) : (Non-separated sheltering mechanism)

- the form stress dominates the contribution of energy input from air to waves at the exponential wave growth stage

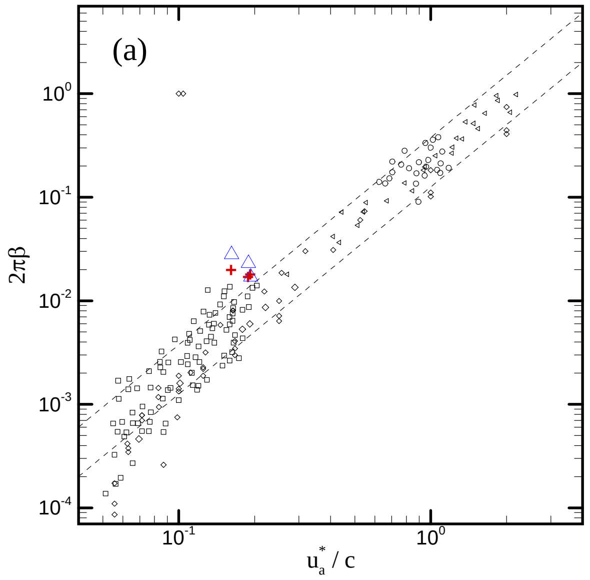
Wave growth rate

$$\beta = \frac{2}{\sigma\, a}\frac{da}{dt} = \frac{1}{\sigma\, E}\frac{dE}{dt}$$

$$\longrightarrow \quad \beta = \frac{1}{\sigma\, E}\frac{dE}{dt} = \frac{2}{\rho_w}\frac{D_p}{(ak)^2}\left(\frac{1}{c}\right)^2$$

where

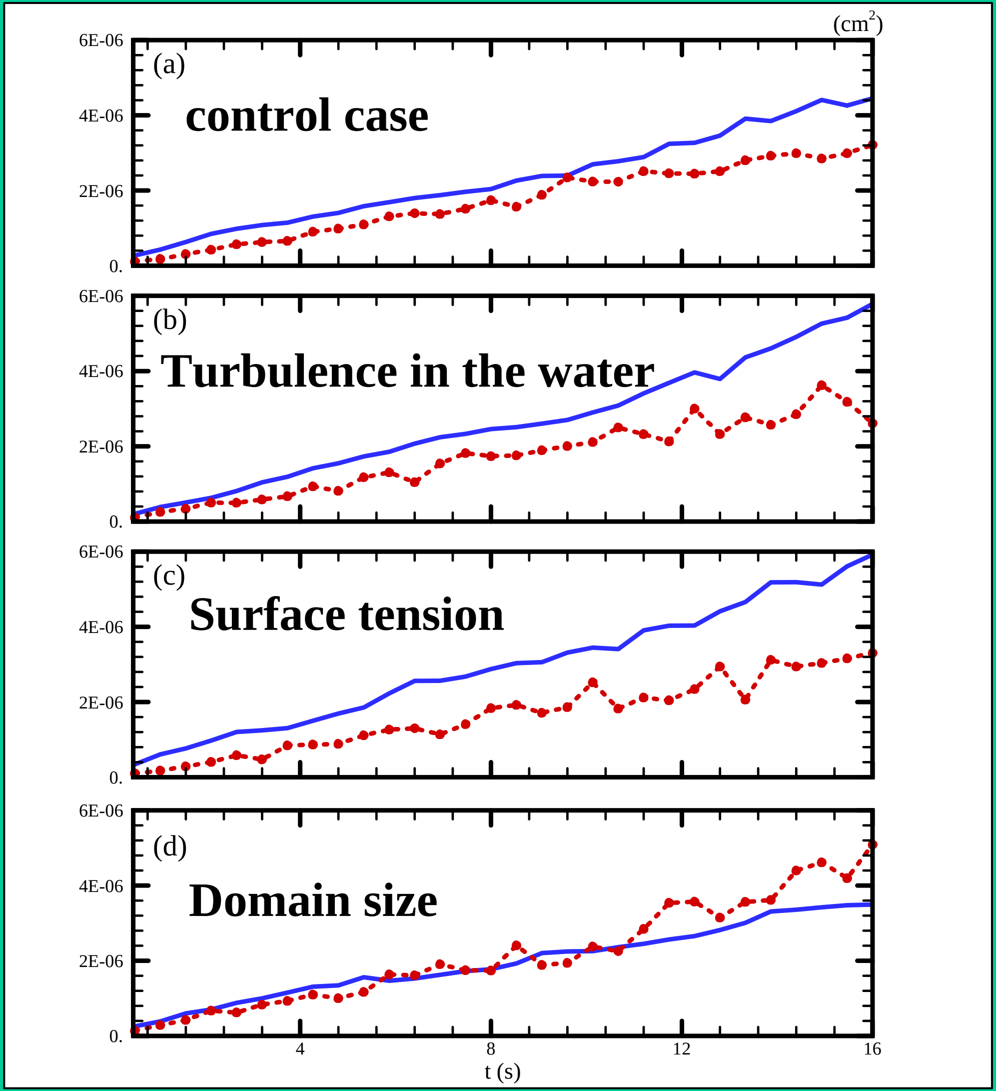
$$D_p = \frac{1}{L_x L_y}\int_0^{L_y}\int_0^{L_x} p'_a \frac{\partial \eta}{\partial x}\, dx\, dy \quad ; \qquad E = 0.5\rho_w a^2 c^2 k$$



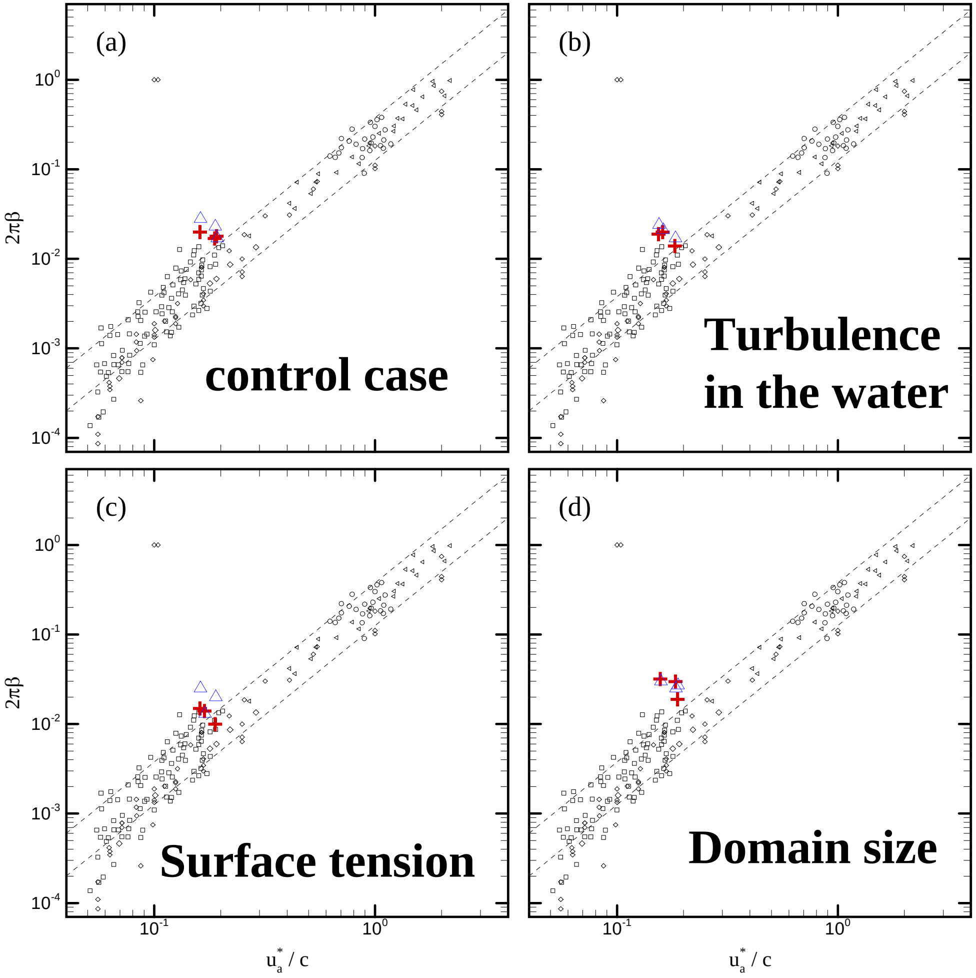
What influences wave growth?

1. **Turbulence in the water**
2. **Surface tension**
3. **Domain size**

Sensitivity Tests



Sensitivity Tests



Summary

- A new air- water coupled model is developed
- The initial wind-wave generation processes is simulated
- The characteristics of flow fields are different at early and late stages
- Wave growth types : linear & exponential
- The wavelengths found here (8-12 cm) are close to those found in laboratory at low wind speed.
- Some of the simulated wave growth rates are close to previous studies' results, but some of them are about 1~3 times larger than their prediction or measurements.

Thank you